Syllabus overview

This book covers the whole syllabus for the DP Mathematics: analysis and approaches SL course. Here is an overview of the syllabus content covered in each chapter.

1 From patterns to generalizations: sequences and series

Syllabus reference	Syllabus content
SL1.2*	Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.
SL1.3*	Geometric sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of geometric sequences. Applications.
SL1.4*	Financial applications of geometric sequences and series: compound interest and annual depreciation.
SL1.6	Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity.
SL1.8	Sum of infinite convergent geometric sequences.
SL1.9	The binomial theorem: expansion of $(a+b)^n, n \in \mathbb{N}$.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

2 Representing relationships: introducing functions

Syllabus reference	Syllabus content
SL2.2*	Concept of a function, domain, range and graph. Function notation, for example $f(x)$, $v(t)$, $C(n)$. The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.
SL2.3*	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.5	Composite functions. Identity function. Finding the inverse function $f^{-1}(x)$.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

3 Modelling relationships: linear and quadratic functions

Syllabus reference	Syllabus content
SL2.1*	Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients, m_1 and m_2 Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.
SL2.4*	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL2.6	The quadratic function $f(x) = ax^2 + bx + c$: its graph, y-intercept 0, c. Axis of symmetry. The form $f(x) = a(x - p)(x - q)$, x intercepts $(p, 0)$ and $(q, 0)$. The form $f(x) = a(x - h)^2 + k$, vertex (h, k).
SL2.7	Solution of quadratic equations and inequalities. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.
SL2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.
SL2.11	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x) - a$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$. Composite transformations.
SL4.4*	Linear correlation of bivariate data. Pearson's product−moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

4 Equivalent representations: rational functions

Syllabus reference	Syllabus content
SL2.3*	The graph of a function; its equation $y = f(x)$. Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4*	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL2.8	The reciprocal function $f(x) = \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature. Rational functions of the form $f(x) = \frac{ax+b}{cx+d}$ and their graphs. Equations of vertical and horizontal asymptotes.
SL2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.
SL2.11	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x) - a$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$. Composite transformations.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

5 Measuring change: differentiation

Syllabus reference	Syllabus content
SL5.1*	Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change.
SL5.2*	Increasing and decreasing functions. Graphical interpretation of $f'(x)>0, f'(x)=0, f'(x)<0$.
SL5.3*	Derivative of $f(x)=ax^n$ $f'(x)=anx^{n-1}$, $n \in \mathbb{Z}$ The derivative of functions of the form $f(x)=ax^n\_bx^{n-1}$ . . . where all exponents are integers.
SL5.4*	Tangents and normals at a given point, and their equations.
SL5.6	Derivative of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, e^x and $\ln x$. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product and quotient rules.
SL5.7	The second derivative. Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''.
SL5.8	Local maximum and minimum points. Testing for maximum and minimum. Optimization. Points of inflexion with zero and non-zero gradients.
SL5.9	Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

6 Representing data: statistics for univariate data

Syllabus reference	Syllabus content
SL4.1*	Concepts of population, sample, random sample, discrete and continuous data. Reliability of data sources and bias in sampling. Interpretation of outliers. Sampling techniques and their effectiveness.
SL4.2*	Presentation of data (discrete and continuous): frequency distributions (tables). Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). Production and understanding of box and whisker diagrams.
SL4.3*	Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Effect of constant changes on the original data. Quartiles of discrete data.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

7 Modelling relationships between two data sets: statistics for bivariate data

Syllabus reference	Syllabus content
SL4.4*	Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r. Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.
SL4.10	Equation of the regression line of x on y. Use of the equation for prediction purposes.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

8 Quantifying randomness: probability

Syllabus reference	Syllabus content
SL4.5*	Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $\mathrm{P}(A) = \frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Expected number of occurrences.
SL4.6*	Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities. Combined events: $\mathrm{P}(A \cup B) = P(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. Mutually exclusive events: $\mathrm{P}(A \cap B) = 0$ Conditional probability: $\mathrm{P}(AB) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ Independent events: $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$.
SL4.11	Formal definition and use of the formulae: $\mathrm{P}(A\|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ for conditional probabilities, and $\mathrm{P}(A\|B) = \mathrm{P}(A) = \mathrm{P}(A\|B')$ for independent events.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

9 Representing equivalent quantities: exponentials and logarithms

Syllabus reference	Syllabus content
SL1.5*	Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.7	Laws of exponents with rational exponents. Laws of logarithms. $\log_a xy = \log_a x + \log_a y$. $\log_a \frac{x}{y} = \log_a x - \log_a y$. $\log_a x^m = m\log_a x$ for $a, x, y > 0$. Change of base of a logarithm. $\log_a x = \frac{\log_b x}{\log_b a}$ for $a, b, x > 0$. Solving exponential equations, including using logarithms.
SL2.9	Exponential functions and their graphs: $f(x) = a^x, a > 0, f(x) = \mathrm{e}^x$. Logarithmic functions and their graphs: $f(x) = \log_a x, x > 0, f(x) = \ln x, x > 0$.
SL2.10	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Applications of graphing skills and solving equations that relate to real-life situations.
SL5.6	Derivative of x^n $(n \in \mathbb{Q})$, sin x, cos x, e^x and ln x. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product and quotient rules.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

10 From approximation to generalization: integration

Syllabus reference	Syllabus content
SL5.5*	Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1}$+..., where $n \in \mathbb{Z}$, $n \neq -1$ Anti-differentiation with a boundary condition to determine the constant term. Definite integrals using technology. Areas between a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.
SL5.9	Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.
SL5.10	Indefinite integral of x^n $(x \in \mathbb{Q})$, sin x, cos x, $\frac{1}{x}$ and e^x. The composites of any of these with the linear function $ax + b$. Integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\int kg'(x)f(g(x))dx$.
SL5.11	Definite integrals, including analytical approach. Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x)$can be positive or negative, without the use of technology. Areas between curves.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

11 Relationships in space: geometry and trigonometry in 2D and 3D

Syllabus reference	Syllabus content
SL3.1*	The distance between two points in three- dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.
SL3.2*	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab \sin C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Area of a triangle as $\frac{1}{2}ab \sin C$.
SL3.3*	Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

12 Periodic relationships: trigonometric functions

Syllabus reference	Syllabus content
SL3.4	The circle: radian measure of angles; length of an arc; area of a sector.
SL3.5	Definition of $\tan\theta$ as $\frac{sin\theta}{\cos\theta}$.
SL3.6	Exact values of trigonometric ratios of 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and their multiples. Extension of the sine rule to the ambiguous case.
SL3.7	The circular functions sin x, cos x, and tan x; amplitude, their periodic nature, and their graphs. Composite functions of the form $f(x) = a\sin(b(x+c)+d)$. Transformations. Real-life contexts.
SL3.8	Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in sin x, cos x, or tan x.

13 Modelling change: more calculus

Syllabus reference	Syllabus content
SL5.6	Derivative of x^n $(n \in \mathbb{Q})$, sin x, cos x, e^x and ln x. Differentiation of a sum and a multiple of these functions. The chain rule for composite functions. The product and quotient rules.
SL5.8	Local maximum and minimum points. Testing for maximum and minimum. Optimization. Points of inflexion with zero and non-zero gradients.
SL5.9	Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled.
SL5.10	Indefinite integral of x^n $(x \in \mathbb{Q})$, sin x, cos x, $\frac{1}{x}$ and e^x. The composites of any of these with the linear function $ax + b$. Integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\int kg'(x)f(g(x))\mathrm{d}x$.

14 Valid comparisons and informed decisions: probability distributions

Syllabus reference	Syllabus content
SL4.3*	Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Effect of constant changes on the original data. Quartiles of discrete data.
SL4.7*	Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data. Applications.
SL4.8*	Binomial distribution. Mean and variance of the binomial distribution.
SL4.9*	The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. Normal probability calculations. Inverse normal calculations
SL4.12	Standardization of normal variables (z-values). Inverse normal calculations where mean and standard deviation are unknown.

**Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*