IB Semester One Final Exam Book Solutions

Exercise 8D

5 Let x be Kelly's score on the fifth test. To average 84

$84 = \frac{95+82+76+88+x}{5}$

$84 \times 5 = 341 + x$

$420 = 341 + x$

$x = 420 - 341$

$x = 79$

Kelly must score 79 on the next test.

6 Total mass of 11 players = $11 \times 80.3 = 883.3$ kg

$81.2 = \frac{883.3+x}{12}$

$81.2 \times 12 = 883.3 + x$

$974.4 = 883.3 + x$

$x = 974.4 - 883.3$

$x = 91.1$ kg

The new player has mass 91.1 kg

7 Let x be the distance they travel on sixth day.

$250 = \frac{220+300+210+275+240+x}{6}$

$250 \times 6 = 1245 + x$

$1500 = 1245 + x$

$x = 1500 - 1245$

$x = 255$

They must travel 255 km on the last day.

8 Total number of shots = $8 \times 71 = 568$.

9 After 8 matches, the total points scored is

$8 \times 27 = 216$

After 11 matches the total points scored is

$11 \times 29 = 319$

319–216 = 103 points scored in the last 3 matches

10 Billy's total = $12 \times 310 = \$3720$

Jean's total = $13 \times 320 = \$4160$

Billy's total + Jean's total = \$7880

Mean = $\frac{7880}{25} = \$315.20$

Exercise 10F

3 **a** 0.907

b The stopping distance increases, as the car gets older.

c Strong positive correlation.

Review exercise ch 10

3 **a** $r = 0.785$ **b** $y = 30.7 + 0.688x$

c IQ = 30.6 + (0.688 × 100) = 99.4. This should be reasonably accurate since the product moment correlation coefficient shows fairly strong correlation.

Review exercise ch 8

1 **a** Mode = 3 as 3 appears the most in the list.

b First write the numbers in ascending order: 1, 2, 3, 3, 5, 6, 7, 8, 10

Median = $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{9+1}{2}\right)^{th} = 5^{th} = 5$.

c Mean = $\frac{1+2+3+3+5+6+7+8+10}{9} = 5$

d Range = 10 - 1 = 9.

Review exercise ch 8

4 **a** Mean = 2.57, median = 2, mode = 1, standard deviation = 1.68 and variance = 2.82.

b Range = 6, lower quartile = 1 and the interquartile range = 3.

Exercise 15J

1 $X \sim N(100, 20^2)$

a $P(X < 130) = 0.933$ normal cdf (-10000, 130, 100, 20)

b $P(X > 90) = 0.691$ normal cdf (90, 10000, 100, 20)

c $P(80 < X < 125) = 0.736$ normal cdf (80, 125, 100, 20)

d $P(X < x) = .90$ invNorm (.9, 100, 20) = \$126

Review exercise ch 15

1 **a** $0.3 + \frac{1}{k} + \frac{2}{k} + 0.1 + 2.1 = 1$

$\frac{3}{k} = 0.5 \quad \therefore k = 6$

b $E(X) = (-2 \times 0.3) + \left(-1 \times \frac{1}{6}\right) + (1 \times 0.1) + (2 \times 0.1)$

$= -\frac{7}{15}$

Review exercise ch 15

2 $X \sim B(8, 0.3)$

a $P(X = 3) = 0.254$ Binompdf (8, .3, 3)

b $P(X \geq 3) = 0.448$ 1 − Binomcdf (8, .3, 2)

c $P(X \leq 2) = 0.552$ Binomcdf (8, .3, 2)

Exercise 12C

4 $\overrightarrow{LM} = \overrightarrow{LN} + \overrightarrow{NM} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$

Exercise 12E

1 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$

Distance $AB = \sqrt{5^2 + (-2)^2}$

$= \sqrt{29}$

≈ 5.39

Exercise 12I

3 (c) $\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} -8 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} = (-8 \times 4) + (2 \times -1) + (2 \times -1)$

$= -32 - 2 - 2 = -36$

$|\mathbf{u}| = \sqrt{8^2 + 2^2 + 2^2} = \sqrt{64 + 4 + 4} = \sqrt{72}$

$|\mathbf{v}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{16 + 1 + 1} = \sqrt{18}$

$|\mathbf{u}||\mathbf{v}| = \sqrt{18 \times 72} = 36 = -\mathbf{u} \cdot \mathbf{v}$

$\Rightarrow$ parallel.

7 (a) $\begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix} = 2 \times 2 + -1 \times 5 = 4 - 5 = -1$

$\left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\left| \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$

$-1 = \sqrt{5}\sqrt{29} \cos\theta$

$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{145}}\right) = 94.8°$

9 (a) $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = -2 - 6 + 12 = 4$

$\left| \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \right| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$

$\left| \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \right| = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = 7$

so $4 = 3 \times 7 \cos\theta$

$\cos\theta = \frac{4}{21}, \theta = 79°$

Exercise 12J

2 (a) Position vectors are $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Line joining the 2 points has direction

$\begin{pmatrix} 3 & -4 \\ -2 & -5 \end{pmatrix} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$ or $\begin{pmatrix} 4 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 7 \end{pmatrix}$

Line is $\mathbf{r} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 7 \end{pmatrix}, t \in \mathbb{R}$.

(c) Position vectors $\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$

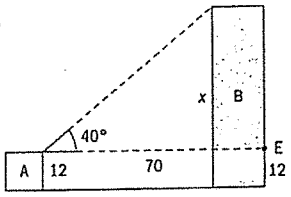
Line joining 2 points has direction

$\begin{pmatrix} 3 & -2 \\ 5 & -(-4) \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \\ -3 \end{pmatrix}$

Line is $\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 9 \\ -3 \end{pmatrix}, t \in \mathbb{R}$.

Exercise 11C

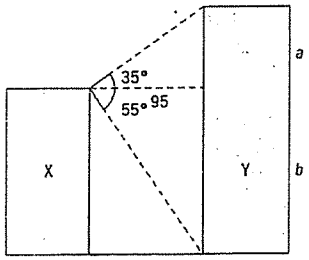
6



$\tan 40 = \frac{x}{70} \rightarrow x = 70 \tan 40 \approx 58.737$

height $= x + 12 \approx 70.7$ m

8



$\tan 35 = \frac{a}{95} \rightarrow a = 95 \tan 35 \approx 66.5197$

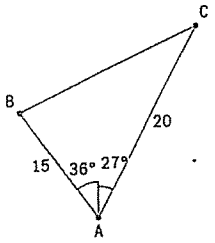
$\tan 55 = \frac{b}{95} \rightarrow b = 95 \tan 55 \approx 135.674$

X is 135.7 m tall, Y is (135.7 + 66.5)

= 202.2 m tall

Exercise 11I

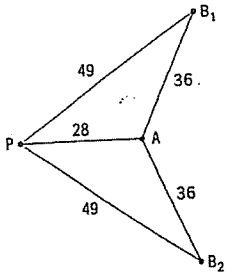
4



$B\hat{A}C = 36 + 27 = 63°$

$BC = \sqrt{15^2 + 20^2 - 2(15)(20)\cos 63} \approx 18.8$ km

5



$\cos A\hat{P}B = \frac{49^2 + 28^2 - 36^2}{2(49)(28)}$

$\rightarrow A\hat{P}B = \cos^{-1}\left(\frac{49^2 + 28^2 - 36^2}{2(49)(28)}\right) \approx 46.5°$

E46.5°N or E46.5°S which is a bearing of 043.5° or 136.5° (since 90 − 46.5 = 43.5 and 90 + 46.5 = 136.5).

Exercise 11M

3 a $\frac{1}{2}(4.5)(4.5)\sin 1.3 \approx 9.76\text{ cm}^2$

b $BC = \sqrt{4.5^2 + 4.5^2 - 2(4.5)(4.5)\cos 1.3} \approx 5.45$ cm

Shaded area = area of circle − area of sector

$= \pi(4.5)^2 - \frac{1.3(4.5)^2}{2}$

$= 50.5\text{ cm}^2$ (3 sf)

5 a Area of $\Delta POQ = \frac{1}{2}(6)(6)\sin 1.25 \approx 17.1\text{ cm}^2$

b $Q\hat{O}R = \cos^{-1}\left(\frac{6^2 + 6^2 - 11.2^2}{2(6)(6)}\right) \approx 2.407$ rad

Area of $\Delta QOR = \frac{1}{2}(6)(6)\sin Q\hat{O}R \approx 12.1\text{ cm}^2$

c $\theta = 2\pi - 1.25 - Q\hat{O}R \approx 2.63$ rad

d $\theta(6) \approx 15.8$ cm

Review exercise ch.11

6 $A\hat{P}B = 170 - 50 = 120°$

$AB = \sqrt{24^2 + 38^2 - 2(24)(38)\cos 120} \approx 54.1$ km

Exercise 13E

4 a $\sin^2 x + \cos^2 x = 1$

$\left(-\frac{1}{8}\right)^2 + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \left(-\frac{1}{8}\right)^2 = \frac{63}{64}$

$\cos x = -\frac{\sqrt{63}}{8}$

$\sin(2x) = 2\sin x \cos x = 2\left(-\frac{1}{8}\right)\left(-\frac{\sqrt{63}}{8}\right) = \frac{\sqrt{63}}{32}$

b $\cos(2x) = 1 - 2\sin^2 x = 1 - 2\left(-\frac{1}{8}\right)^2 = 1 - \frac{1}{32} = \frac{31}{32}$

c $\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{\left(\frac{\sqrt{63}}{32}\right)}{\left(\frac{31}{32}\right)} = \frac{\sqrt{63}}{31}$

d $\sin(4x) = 2\sin(2x)\cos(2x)$

$= 2\left(\frac{\sqrt{63}}{32}\right)\left(\frac{31}{32}\right) = \frac{31\sqrt{63}}{512}$

Exercise 13F

2 d $1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = 30°, 150°$ or $x = -90°$

Review exercise ch.13

4 $\sin 2x + \sin x = 0$

$2\sin x \cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ or $\cos x = -\frac{1}{2}$

$x = 0, \pi$ or $x = \frac{2\pi}{3}$

5 a i amplitude: $a = \frac{11 - 1}{2} = 5$

horizontal shift: $c = 4$

vertical shift: $d = \frac{11 + 1}{2} = 6$

ii $b = \frac{2\pi}{period}$, and the period is 8. $b = \frac{2\pi}{8} = \frac{\pi}{4}$

b $4 < x < 8$

6 a $\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(\frac{2}{5}\right)^2 = 1 \rightarrow \sin^2 x = 1 - \left(\frac{2}{5}\right)^2 = \frac{21}{25}$

$\sin x = \frac{\sqrt{21}}{5}$

b $\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\left(\frac{\sqrt{21}}{5}\right)}{\left(\frac{2}{5}\right)} = \frac{\sqrt{21}}{2}$

c $\sin(2x) = 2\sin x \cos x = 2\left(\frac{\sqrt{21}}{5}\right)\left(\frac{2}{5}\right) = \frac{4\sqrt{21}}{25}$