Review exercise

Paper 1 style questions

1 It is stated that the content of a can of drink is 350 ml. The content of thousands of cans is tested and found to be normally distributed with a mean of 354 ml and a standard deviation of 2.5 ml.

  a Sketch a normal distribution diagram to illustrate this information.

  b Find the probability that a can contains less than 350 ml.

100 cans are chosen at random.

  c Find the expected number of cans that contain less than 350 ml.

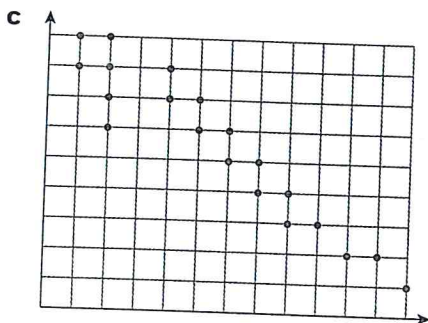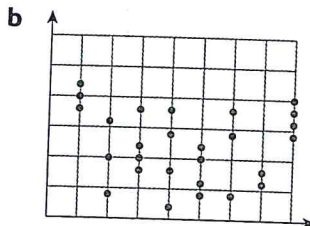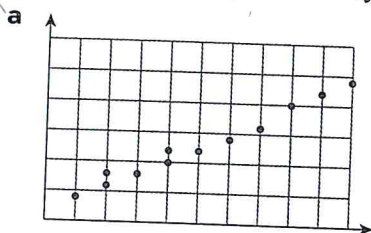*[handwritten: Give proper notation too ← $X \sim N(\mu, 6^2)$]*

2 6000 people were asked how far they lived from their work. The distances were normally distributed with a mean of 4.5 km and a standard deviation of 1.5 km.

  a Find the percentage of people who live between 2 km and 4 km from their work.

  b Find the expected number of people who live less than 1 km from their work.

3 The weights of bags of tomatoes are normally distributed with a mean of 1.03 kg and a standard deviation of 0.02 kg.

  a Find the percentage of bags that weigh more than 1 kg.

It is known that 15% of the bags weigh less than p kg.

  b Find the value of p.

4 For each diagram, state the type of correlation.

  a

  b

  c

9 60 students go ten-pin bowling. They each have one throw with their right hand and one throw with their left. The number of pins knocked down each time is noted. The results are collated in the table.

	0–3	4–7	8–10	Totals
Right hand	8	28	24	60
Left hand	12	30	18	60
Totals	20	58	42	120

A χ^2 test is performed at the 10% significance level.

a State the null hypothesis.

b Write down the number of degrees of freedom.

c Show that the expected number of students who knock down 0–3 pins with their right hand is 10.

~~The χ^2 value is 0.422~~

d Write down the conclusion reached at the 10% significance level.

Give a clear reason for your answer.

10 Erland performs a chi-squared test to see if there is any association between the preparation time for a test (short time, medium time, long time) and the outcome (pass, does not pass). Erland performs this test at the 5% significance level.

non calc

a Write down the null hypothesis.

b Write down the number of degrees of freedom.

The *p*-value for this test is 0.069. And the χ^2_{calc} is 4.21

c What conclusion can Erland make?

Justify your answer.

Table from 12.4 notes

Paper 2 style questions

EXAM-STYLE QUESTIONS

1 The heights of Dutch men are normally distributed with a mean of 181 cm and a standard deviation of 9 cm.

a Sketch a normal distribution diagram to illustrate this information.

b Find the probability that a man chosen at random has a height less than 175 cm.

c Find the probability that a man chosen at random has a height between 172 cm and 192 cm.

Sixty men are measured.

d Find the expected number of men with a height greater than 195 cm.

It is known that 5% of the men have a height less than k cm.

e Find the value of k.

non-calc

2 The weights of bags of sweets are normally distributed with a mean of 253 g and a standard deviation of 3 g.
   a Sketch a diagram to illustrate this information clearly.
   b Find the percentage of bags expected to weigh less than 250 g. *Find the percentage of bags weighing between 253g and 262g*

   Three hunderd bags are weighed.

   c Find the expected number of bags weighing more than ~~350g~~ *256g*.

EXAM-STYLE QUESTIONS

8 Members of a certain club are required to register for one of three games: badminton, table tennis or darts.
   The number of club members of each gender choosing each game in a particular year is shown in the table.

	Badminton	Table tennis	Darts
Male	37	16	28
Female	32	10	19

   Use a chi-squared test, at the 5% significance level, to test whether choice of game is independent of gender. State clearly the null and alternative hypotheses, the expected values and the number of degrees of freedom. *Use the table to look up the cv.*

use p + χ^2_{calc}

use table from 12.4 notes

9 For his Mathematical Studies Project a student gave his classmates a questionnaire to find out which extra-curricular activity was the most popular. The results are given in the table below, classified by gender.

	Reading	Surfing	Skating	
Female	22	16	22	(60)
Male	14	18	8	(40)
	(36)	(34)	(30)	

   The table below shows the expected values.

	Reading	Surfing	Skating
Female	p	20.4	18
Male	q	r	12

   a Calculate the values of p, q and r.

   The chi-squared test, at the 10% level of significance, is used to determine whether the extra-curricular activity is independent of gender.

   b i State a suitable null hypothesis.
      ii Show that the number of degrees of freedom is 2.

   The critical value is 4.605.

   c Write down the chi-squared statistic.

   d Do you accept the null hypothesis? Explain your answer.

10 A company conducted a survey to determine whether position in upper management was independent of gender. The results of this survey are tabulated below.

	Managers	Junior executives	Senior executives	Totals
Male	135	90	75	300
Female	45	130	25	200
Totals	180	220	100	500

The table below shows the expected number of males and females at each level, if they were represented proportionally to the total number of males and females employed.

	Managers	Junior executives	Senior executives	Totals
Male	a	c	60	300
Female	b	d	40	200
Totals	180	220	100	500

a i Show that the expected number of male managers (a) is 108.

  ii Hence, write down the values of b, c and d.

b Write suitable null and alternative hypotheses for these data.

c i Find the chi-squared value.

  ii Write down the number of degrees of freedom.

  iii Given that the critical value is 5.991, what conclusions can be drawn regarding gender and position in upper management?

11 In the small town of Schiedam, population 8000, an election was held. The results were as follows.

	Urban voters	Rural voters
Candidate A	1950	1730
Candidate B	1830	1360
Candidate C	500	630

In **a–d** below, use a chi-squared test, at the 1% significance level, to decide whether the choice of candidate depends on where the voter lives.

H_0: The choice of candidate is independent of where the voter lives.

a Write down the alternative hypothesis.

b Show that the expected number of rural voters for candidate A is 1711.

c i Calculate the chi-squared value.

  ii Write down the number of degrees of freedom.

The critical value is 9.21.

d i State your conclusion.

  ii Explain why you reached your conclusion.

12 This table of observed results gives the number of candidates taking a Mathematics examination classified by gender and grade obtained.

	6 or 7	4 or 5	1, 2 or 3	Totals
Males	34	50	6	90
Females	40	60	10	110
Totals	74	110	16	200

The question posed is whether gender and grade obtained are independent.

a Show that the expected number of males achieving a grade of 4 or 5 is 49.5.

A chi-squared test is set up at the 5% significance level.

b i State the null hypothesis.

 ii State the number of degrees of freedom.

 **iii** Write down the chi-squared value.

The critical value is 5.991.

c What can you say about gender and grade obtained?

CHAPTER 5 SUMMARY
The normal distribution

- The **normal distribution** is the most important continuous distribution in statistics. It has these properties:
  - It is a bell-shaped curve.
  - It is symmetrical about the mean, μ. (The mean, the mode and the median all have the same value.)
  - The x-axis is an asymptote to the curve.
  - The total area under the curve is 1 (or 100%).
  - 50% of the area is to the left of the mean and 50% to the right.
  - Approximately 68% of the area is within 1 standard deviation, σ, of the mean.
  - Approximately 95% of the area is within 2 standard deviations of the mean.
  - Approximately 99% of the area is within 3 standard deviations of the mean.
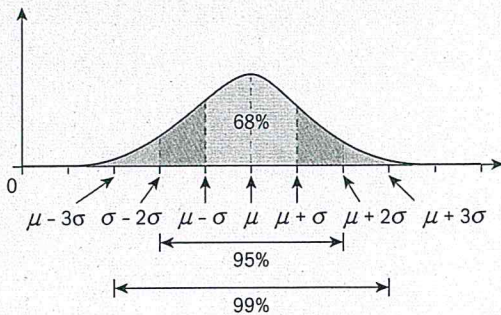
- The **expected value** is found by multiplying the number in the sample by the probability.

non
Calc

Oral tests are conducted by three examiners A, B and C separately. The results of the examination are classified as Credit, Pass or Fail. A χ^2 test is applied to the data collected in order to test whether or not the examiners differ in their standard of awards.

(a)   State the null hypothesis, H_0, for this data.

(b)   Write down the number of degrees of freedom.

Of the 135 students who sit the exam, 30 get Credit and 45 are tested by examiner A.

(c)   Calculate the expected number of students who get a Credit and are tested by examiner A.

Using a 5% level of significance, the p-value is found to be 0.0327 correct to 3 s.f.

(d)   State whether H_0 should be accepted. Justify your answer.

(Total 6 marks)

Review exercise

1 The table shows the probability distribution of a
discrete random variable X.
 a Find the value of k.
 b Find the expected value of X.

x	-2	-1	0	1	2
$P(X = x)$	0.3	$\dfrac{1}{k}$	$\dfrac{2}{k}$	0.1	0.1

2 The probability distribution of a discrete random variable X is
defined by $P(X = x) = cx(6 - x)$, $x = 1, 2, 3, 4, 5$.
 a Find the value of c.   **b** Find $E(X)$.

3 In a game a player rolls a biased tetrahedral (four-faced)
die. The probability of each possible score is shown.
Find the probability of a total score of six after two rolls.

Score	1	2	3	4
Probability	$\dfrac{1}{4}$	$\dfrac{1}{4}$	$\dfrac{1}{8}$	x

4 A game involves spinning two spinners. One is numbered 1, 2, 3, 4.
The other is numbered 2, 2, 4, 4. Each spinner is spun once and the
number on each is recorded.
Let P be the product of the numbers on the spinners
 a Write down all the possible values for P.
 b Find the probability of each value of P.
 c What is the expected value of P?
 d A mathematician determines the amount of pocket money to
  give his son each week by getting him to play the game on
  Monday morning. The son spins and if the product is greater
  than 10 then he gets £10. Otherwise he gets £5. How much in
  total will the boy expect to get after 10 weeks of playing the game?

5 In a train, $\dfrac{1}{3}$ of the passengers are listening to music. Five passengers are chosen
at random. Find the probability that exactly three are listening to music.

6 When a boy plays a game at a fair, the probability that he wins a prize
is 0.1. He plays the game twice. Let X denote the total number of prizes
that he wins. Assuming that the games are independent, find $E(X)$.

7 Let X be normally distributed with mean 75 and standard deviation 5.
 a Given that $P(X < 65) = P(X > a)$, find the value of a.
 b Given that $P(65 < X < a) = 0.954$, Find $P(X > a)$.

Section 12.1