Radicals:

To simplify means that 1) no radicand has a perfect square factor and
2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both the numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
multiply the numerator and the denominator by the ***conjugate*** of the denominator
The ***conjugate*** of $3 + \sqrt{2}$ is $3 - \sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\sqrt{60} \bullet \sqrt{105}$
7. $\left(\sqrt{5} - \sqrt{6}\right)\left(\sqrt{5} + \sqrt{2}\right)$

Rationalize.

8. $\frac{1}{\sqrt{2}}$
9. $\frac{2}{\sqrt{3}}$
10. $\frac{3}{2 - \sqrt{5}}$

Complex Numbers:

Form of complex number - $a+bi$

Where a is the "real" is part and bi is the "imaginary" part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\cdot\sqrt{5}$ Pull out $\sqrt{-1}$

$= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5}\cdot i\sqrt{5}$ List twice

$= i^2\sqrt{25}$ Simplify

$= (-1)(5) = -5$ Substitute

- Treat i like any other variable when +, -, ×, or ÷ (but always simplify $i^2=-1$)

Example: $2i(3+i) = 2(3i)+2i(i)$ Distribute

$= 6i+2i^2$ Simplify

$= 6i+2(-1)$ Make substitution

$= -2+6i$ Simplify and rewrite in complex form

- Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator **RATIONALIZE!!**

Simplify.

9. $\sqrt{-49}$

10. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

14. $(2+3i)(5-4i)$

Rationalize.

15. $\frac{1+6i}{5i}$

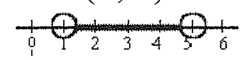
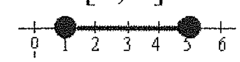
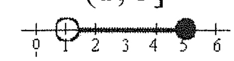
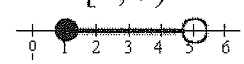
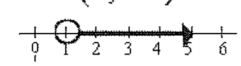
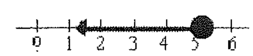
16. $\frac{1+3i}{1+2i}$

Interval Notation

An interval is a connected subset of numbers. Interval notation is an alternative to expressing your answer as an inequality. Unless specified otherwise, we will be working with real numbers.

When using interval notation, the symbol (means "not included" or "open" and [means "included" or "closed".

The chart below will show you all of the possible ways of utilizing interval notation

Interval Notation: (description)	(diagram)
Open Interval: (a, b) is interpreted as $a < x < b$ where the endpoints are NOT included. (While this notation resembles an ordered pair, in this context it refers to the interval upon which you are working.)	$(1, 5)$ 
Closed Interval: $[a, b]$ is interpreted as $a \leq x \leq b$ where the endpoints are included.	$[1, 5]$ 
Half-Open Interval: $(a, b]$ is interpreted as $a < x \leq b$ where a is not included, but b is included.	$(1, 5]$ 
Half-Open Interval: $[a, b)$ is interpreted as $a \leq x < b$ where a is included, but b is not included.	$[1, 5)$ 
Non-ending Interval: (a, ∞) is interpreted as $x > a$ where a is not included and infinity is always expressed as being "open" (not included).	$(1, \infty)$ 
Non-ending Interval: $(-\infty, b]$ is interpreted as $x \leq b$ where b is included and again, infinity is always expressed as being "open" (not included).	$(-\infty, 5]$ 

Write the interval notation for the inequalities given:

17. $x \geq 4$

18. $-3 < x < 2$

19. $x < -2$

20. $-5 < x \leq 3$

21. $6 < x$

22. $-10 \leq x \leq -5$

Equations of Lines:

Slope intercept form: $y = mx + b$	**Vertical line:** $x = c$ (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	**Horizontal line:** $y = c$ (slope is 0)
Standard Form: $Ax + By = C$	**Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$.

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

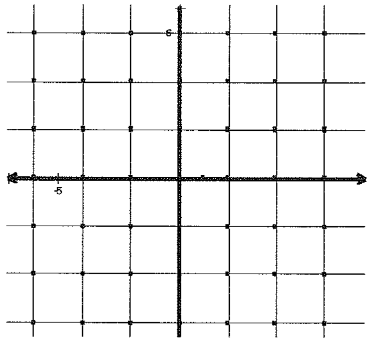
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

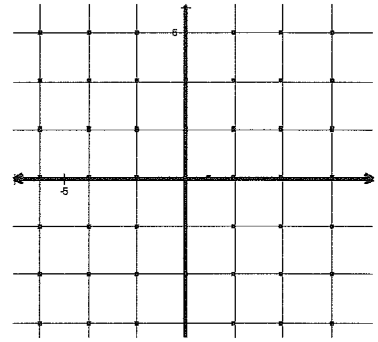
Graphing:

Graph each function, inequality, and / or system.

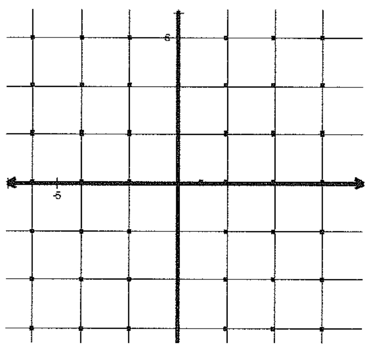
29. $3x - 4y = 12$



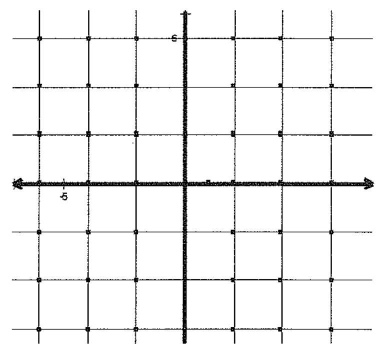
30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$



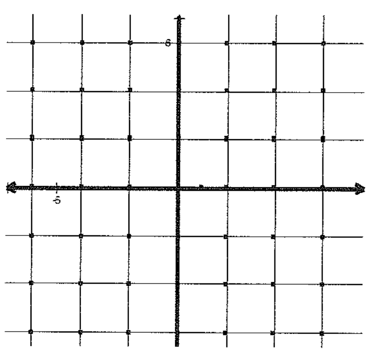
31. $y < -4x - 2$



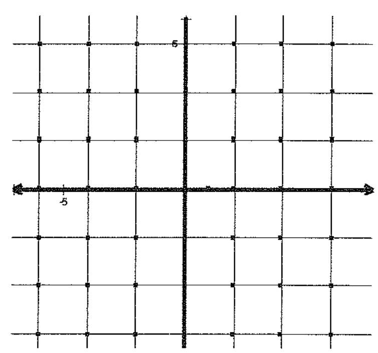
32. $y + 2 = |x + 1|$



33. $y > |x| - 1$



34. $y + 4 = (x - 1)^2$



Vertex: ____________________

x-intercept(s): ____________________

y-intercept(s): ____________________

Systems of Equations:

$3x + y = 6$
$2x - 2y = 4$

Substitution:	**Elimination:**
Solve 1 equation for 1 variable.	Find opposite coefficients for 1 variable.
Rearrange.	Multiply equation(s) by constant(s).
Plug into 2nd equation.	Add equations together (lose 1 variable).
Solve for the other variable.	Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	solve 1st equation for y	$6x + 2y = 12$	multiply 1st equation by 2
$2x - 2(6 - 3x) = 4$	plug into 2nd equation	$2x - 2y = 4$	coefficients of y are opposite
$2x - 12 + 6x = 4$	distribute	$8x = 16$	add
$8x = 16$	simplify	$x = 2$	simplify
$x = 2$			

Plug $x = 2$ back into original

$$3(2) + y = 6$$
$$6 + y = 6$$
$$y = 0$$

Solve each system of equations. Use any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\frac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add / subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*
$=8x-3y+6-6y-4x+9$ *Combine terms with similar variables.*
$=8x-4x-3y-6y+6+9$
$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiplying two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, then last terms.*
$=3x^2+12x-2x-8$ *Combine like terms.*
$=3x^2+10x-8$

Multiply.

48. $(3a+1)(a-2)$

49. $(s+3)(s-3)$

50. $(c-5)^2$

51. $(5x+7y)(5x-7y)$

Factoring

Examples: Factor.

GCF: $4x^3y+6x^2y-2x$

$=2x(2x^2y+3xy-1)$

Difference of 2 Squares: $4x^2-9$

$=(2x)^2-(3)^2$

$=(2x-3)(2x+3)$

Product/Sum: $x^2+2x-15$

(or reverse FOIL) $=(x+5)(x-3)$

x (-15)	+ (2)
-1, 15	14
1, -15	-14
-5, 3	-2
5, -3	+2

By Grouping: x^3-4x^2+2x-8

$=(x^3-4x^2)+(2x-8)$

$=x^2(x-4)+2(x-4)$

$=(x-4)(x^2+2)$

- *Use ( )+( ) to group*
- *Factor out the GCF of each group*
- *Factor out the common (_) out of each term*

Master Product: $2x^2-7x+3$

(or just guess and check)

x 6	+ (-7)
2, 3	5
-2, -3	-5
1, 6	7
-1, -6	-7

- *In* ax^2+bx+c *form, multiply a x c*
- *Find 2 #'s that multiply to axc and add to b*

$= 2x^2\overbrace{-1x-6x}^{-7x}+3$

- *Replace the middle term*

$=(2x^2-1x)+(-6x+3)$

$=x(2x-1)+-3(2x-1)$

$=(2x-1)(x-3)$

- *Factor by grouping.*

Practice: Factor the following completely.

52. $5a^2b+10ab^3$

53. x^2-25

54. x^2+6x

55. $1-9x^2$

56. $6x^3-9x^2+2x-3$

57. $5x^2-20$

58. $x^2-8x+15$

59. $5x^2-7x+2$

60. $2x^2-2x-24$

61. $6x^2-x-1$

Solving Quadratic Equations

***Examples:* Solve the following quadratic equations.**

a.) Solve by factoring:

$$x^2+2x-15=0$$
$$(x-3)(x+5)=0$$
$$x-3=0 \qquad x+5=0$$
$$\boxed{x=3 \qquad x=-5}$$

- *Factor the equation*
- *Set each factor = to 0.*
- *Solve for x.*

b.) Solve by using the quadratic equation:

$$2x^2-3x-6=3$$
$$2x^2-3x-9=0 \qquad a=2, b=-3, c=-9$$
$$x=\frac{+3\pm\sqrt{(-3)^2-4(2)(-9)}}{2(2)}$$
$$x=\frac{3\pm\sqrt{81}}{4}$$
$$x=\frac{3+9}{4}=\frac{12}{4}=3 \qquad x=\frac{3-9}{4}=\frac{-6}{4}=\frac{-3}{2} \qquad \boxed{x=3, \frac{-3}{2}}$$

- *Set up in $ax^2+bx=c=0$ form. and determine a,b,c*
- *Substitute into the quadratic equation:*

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- *Simplify.*

c.) Solve by using the quadratic equation, simplify the radicals:

$$x^2-4x=8$$
$$x^2-4x-8=0 \qquad a=1, b=-4, c=-8$$
$$x=\frac{+4\pm\sqrt{(-4)^2-4(1)(-8)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{48}}{2} \longrightarrow x=\frac{4\pm4\sqrt{3}}{2} \longrightarrow \boxed{x=2\pm2\sqrt{3}}$$

Practice: *Solve.*

62. $x^2+7x+12=0$

63. $5x^2=10x$

64. $2x^2+x=15$

65. $-x^2+2x+10=0$

66. $x^2-16=0$

67. $2x^2-3x-3=0$

Properties of Real Numbers

Real Numbers All real numbers can be classified as either rational or irrational. The set of rational numbers includes several subsets: natural numbers, whole numbers, and integers.

R	real numbers	{all rationals and irrationals}
Q	rational numbers	{all numbers that can be represented in the form $\frac{m}{n}$, where m and n are integers and n is not equal to 0}
I	irrational numbers	{all nonterminating, nonrepeating decimals}
Z	integers	{..., −3, −2, −1, 0, 1, 2, 3, ...}
W	whole numbers	{0, 1, 2, 3, 4, 5, 6, 7, 8, ...}
N	natural numbers	{1, 2, 3, 4, 5, 6, 7, 8, 9, ...}

Example **Name the sets of numbers to which each number belongs.**

a. $-\frac{11}{3}$ rationals (Q), reals (R)

b. $\sqrt{25}$

$\sqrt{25} = 5$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

Exercises

Name the sets of numbers to which each number belongs.

68. π

69. 192.0005

70. 73

71. $-\sqrt{81}$

72. −0.02

73. $\frac{\sqrt{25}}{2}$

74. $33.\overline{3}$

75. $\sqrt{42}$

76. −1

77. $\frac{\sqrt{5}}{2}$

To evaluate a function for a given value, simply plug the value into the function for x.

Evaluate each function for the given value.

78. $f(x) = x^2 - 6x + 2$

$f(3) =$ ___________

79. $g(x) = 6x - 7$

$g(x + h) =$ ___________

80. $f(x) = 3x^2 - 4$

$5[f(x + 2)] =$ ___________

Composition of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find *f(g(x))*.

$$f(g(x)) = f(x-4)$$
$$= 2(x-4)^2+1$$
$$= 2(x^2-8x+16)+1$$
$$= 2x^2-16x+32+1$$
$$f(g(x)) = 2x^2-16x+33$$

Suppose $f(x)=2x,\ g(x)=3x-2,\ and\ h(x)=x^2-4$ **. Find the following:**

81. $f[g(2)] =$ ______

82. $f[g(x)] =$ ______

83. $f[h(3)] =$ ______

84. $g[f(x)] =$ ______

Domain and Range

Given a function y = f(x), the Domain of the function is the set of permissible inputs (x-values) and the Range is the set of resulting outputs (y-values). Domains can be found algebraically; ranges are often found graphically. Domains and Ranges are sets. Therefore, you must use proper set notation.

When finding the domain of a function, ask yourself what values can't be used. Your domain is everything else. There are simple basic rules to consider:

* The domain of all polynomial functions is the set of real numbers.
* Square root functions cannot contain a negative underneath the radical. Set the expression under the radical greater than or equal to zero and solve for the variable. This will be your domain.
* Rational functions cannot have zeros in the denominator. Determine which values of the input cause the denominator to equal zero, and set your domain to be everything else.

Examples: Find the domain of the function algebraically

1. $f(x)=\dfrac{4}{x-2}$ Domain: $x \neq 2$

2. $f(x)=\dfrac{3}{x^2-4x}=\dfrac{3}{x(x-4)}$ Domain: $x \neq 0, 4$

3. $f(x)=\dfrac{x}{x^2-9}=\dfrac{x}{(x+3)(x-3)}$ Domain: $x \neq -3, 3$

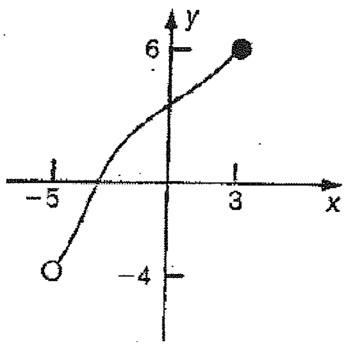
4. $f(x)=\sqrt{x-9}$ Domain: $x \geq 9$

Domain and Range Continued

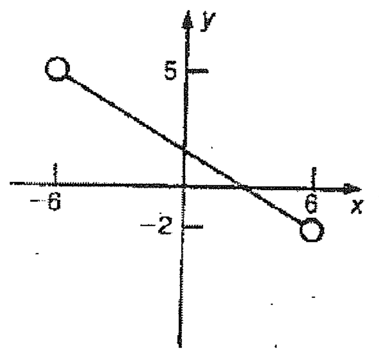
Identifying the domain and range from the graph is easy. For domain values, take the x-values from left to right. For range values, take the y-values from bottom to top.

Examples:

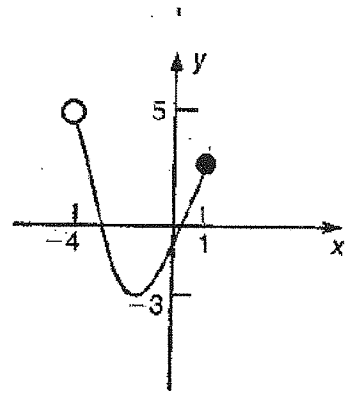
1. Domain: $-5 < x \le 3$
   Range: $-4 < y \le 6$



Domain: $-4 < x \le 1$
Range: $-3 \le y < 5$



Domain: $-6 < x < 6$
Range: $-2 < y < 5$



Practice: Identify the domain for each function

85. $f(x) = x^2 + 3x - 4$

86. $g(x) = \dfrac{x-1}{3x+2}$

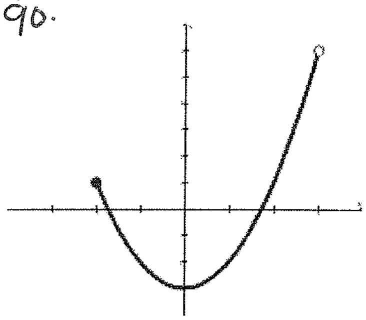
87. $h(x) = \sqrt{2x-4}$

88. $g(x) = \sqrt{5-x}$

89. $f(x) = \dfrac{3}{x^2 - x - 6}$

Practice: Identify the domain and range of each function

90.



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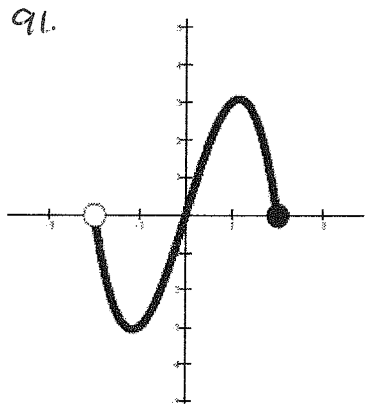

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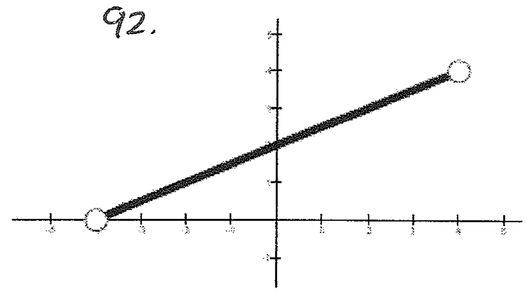

Rational Algebraic Expressions:

Multiplying and Dividing.
Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX:

$$\frac{x^2+10x+21}{5-4x-x^2}\bullet\frac{x^2+2x-15}{x^3+4x^2-21x}$$ *Factor everything completely.*

$$=\frac{(x+7)(x+3)}{(5+x)(1-x)}\bullet\frac{(x+5)(x-3)}{x(x-3)(x+7)}$$ *Cancel out common factors* in *the top and bottom.*

$$=\frac{(x+3)}{x(1-x)}$$ *Simplify.*

Simplify.

93. $\frac{5z^3+z^2-z}{3z}$

94. $\frac{m^2-25}{m^2+5m}$

95. $\frac{10r^5}{21s^2}\bullet\frac{3s}{5r^3}$

96. $\frac{a^2-5a+6}{a+4}\bullet\frac{3a+12}{a-2}$

97. $\frac{6d-9}{5d+1}\div\frac{6-13d+6d^2}{15d^2-7d-2}$

Addition and Subtraction.

First, find the least common denominator.

Write each fraction with the LCD.

Add / subtract numerators as indicated and leave the denominators as they are.

EX: $\dfrac{3x+1}{x^2+2x}+\dfrac{5x-4}{2x+4}$ *Factor denominator completely.*

$=\dfrac{3x+1}{x(x+2)}+\dfrac{5x-4}{2(x+2)}$ *Find LCD* $(2x)(x+2)$

$=\dfrac{2(3x+1)}{2x(x+2)}+\dfrac{x(5x-4)}{2x(x+2)}$ *Rewrite each fraction with the LCD as the denominator.*

$=\dfrac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$=\dfrac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

98. $\dfrac{2x}{5}-\dfrac{x}{3}$

99. $\dfrac{b-a}{a^2b}+\dfrac{a+b}{ab^2}$

100. $\dfrac{2-a^2}{a^2+a}+\dfrac{3a+4}{3a+3}$

Complex Fractions.
Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify as you did above

EX:

$$\frac{1+\frac{1}{a}}{\frac{2}{a^2}-1} \qquad \textit{Find LCD}: a^2$$

$$=\frac{\left(1+\frac{1}{a}\right)\cdot a^2}{\left(\frac{2}{a^2}-1\right)\cdot a^2} \qquad \textit{Multiply top and bottom by LCD.}$$

$$=\frac{a^2+a}{2-a^2} \qquad \textit{Factor and simplify if possible.}$$

$$=\frac{a(a+1)}{2-a^2}$$

or

1. Simplify the numerator.
2. Simplify the denominator.
3. Simplify the division problem that remains.

101. $\dfrac{1-\frac{1}{2}}{2+\frac{1}{4}}$

102. $\dfrac{1+\frac{1}{z}}{z+1}$

103. $\dfrac{5+\frac{1}{m}-\frac{6}{m^2}}{\frac{2}{m}-\frac{2}{m^2}}$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of your fractions. Then solve the equation as usual.

EX:

$$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$$ *Find LCD first.* $x(x+2)$

$$x(x+2)\left(\frac{5}{x+2}\right)+x(x+2)\left(\frac{1}{x}\right)=\left(\frac{5}{x}\right)x(x+2)$$ *Multiply each term by the LCD.*

$$5x+1(x+2)=5(x+2)$$ *Simplify and solve.*

$$5x+x+2=5x+10$$

$$6x+2=5x+10$$

$x=8$ ⇐ *Check your answer. Sometimes they do not check!*

Check :

$$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{8}=\frac{5}{8}$$

or cross-multiply!

$$\frac{3}{x+2}=\frac{12}{5x}$$

$15x=12x+24$ $x=8$

Solve each equation. Check your solutions.

104. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

105. $\frac{x+10}{x^2-2}=\frac{4}{x}$

106. $\frac{5}{x-5}=\frac{x}{x-5}-1$

107. $\frac{3x+4}{2x-5}=1$