Section 9.7 Logical Equivalence, Tautologies, and Contradictions

Definition: Logically Equivalent ~ Two statements are logically equivalent if they have the same truth value under all circumstances. For example, an implication and its contrapositive are logically equivalent as well as the inverse and converse of an implication.

Definition: Tautology ~ A statement is called a tautology if it is always true under any circumstances. For example, If P then P. If a statement is a tautology then it is logically valid.

A valid argument is always true (a tautology). An invalid argument is not always true.

Definition: Contradiction ~ A statement is a contradiction if it is always false under any circumstances. For example, If not P then P.

Example: Show that $\neg p \Rightarrow (p \Rightarrow q)$ is a tautology.

p	**q**	**$\neg$p**	**p $\Rightarrow$ *q***	**$\neg$p $\Rightarrow$ (p $\Rightarrow$ q)**
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Since the last column is all true, it is a tautology.

Example:

Part A:

P: It is cold.
Q: The ice is thick.
R: I will go skating.

Write the statement $\mathbf{p \wedge q \Leftrightarrow r}$ in words.

Answer: If and only if it is cold and the ice is thick, then I will go skating.

Example:

Part B:

Use a truth table to test whether the statement is logically valid.

p	**q**	**r**	**$p \wedge q$**	**$p \wedge q \Leftrightarrow r$**
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T
F	F	T	F	F
F	F	F	F	T

Conclusion:

$p \wedge q \Leftrightarrow r$ is not a valid argument. The final column contains both true and false conclusions. To be valid it must be always true. So it is an invalid argument.

Note: Since it is sometimes true, it is not a contradiction.