## Section 9.6 Inverse, Converse, Contrapositive

| Related Conditional Statements |  |  |
| :--- | :---: | :--- |
| Original Statement | $p \Rightarrow q$ | if p , then q |
| Inverse | $\neg p \Rightarrow \neg q$ | if not p , then not q |
| Converse | $q \Rightarrow p$ | if q, then p |
| Contrapositive | $\neg q \Rightarrow \neg p$ | if not q, then not p |

Example: Write the inverse, converse, and contrapositive of the following statement. Then give its truth value.

If it is 8 PM, then the sun has already set.

Inverse:

If it is not 8 PM , then the sun has not already set.

Truth Value $\sim$ False, it could be 10 PM. If so, the sun would have already set.

Converse:

If the sun has already set, then it is 8 PM .

Truth Value $\sim$ False. The sun could have set but it is 7 PM .

Contrapositive:

If the sun has not already set, then it is not 8 PM .

Truth Value $\sim$ True. The sun would already have set by 8 PM.

Example: Write the inverse, converse, and contrapositive of the following statements
a) $P \Rightarrow Q$
b) $P \Rightarrow \neg Q$
c) $(P \vee Q) \Rightarrow P$

Inverse: $\neg P \Rightarrow \neg Q$
Converse: $\mathrm{Q} \Rightarrow P$
Contrapositive: $\neg Q \Rightarrow \neg P$

Inverse: $\neg P \Rightarrow Q$
Converse: $\neg Q \Rightarrow P$
Contrapositive: $\mathrm{Q} \Rightarrow \neg P$

Inverse: $\neg(P \vee Q) \Rightarrow \neg P$
Converse: $P \Rightarrow(P \vee Q)$
Contrapositive: $\neg P \Rightarrow \neg(P \vee Q)$

Example: Write down in words the meaning of $\neg Q \Rightarrow \neg P$
P : It is raining.
Q: I will bring an umbrella.

Answer: If I will not bring an umbrella then it is not raining.

Complete the truth table below

| $\mathbf{p}$ | $\mathbf{q}$ | $\neg \mathbf{p}$ | $\neg \mathbf{q}$ | $\neg \mathbf{q} \Rightarrow \neg \mathbf{p}$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | T |
| T | F | F | T | F |
| F | T | T | F | T |
| F | F | T | T | T |

