## Section 9.5 Truth Tables

Definition: Truth Table ~ A way of organizing the possible combinations of truth values of two or more propositions. They can be used to determine the truth value of compound statements various circumstances.

Use the Truth Table on your resource sheet (minus the last row of explanation) or your formula booklet. The truth table is the only information you are given in the formula booklet.

To use a truth table work from left to right in the table. Then use the columns you have figured out to deduce the truth values of the further columns.

Examples: Complete the truth tables below:

1. $\neg \mathrm{p} \wedge \neg \mathrm{q}$

Remember from Section 9.3... Conjunctions ( $\wedge$ ) are true only when their two conjuncts are true)

| $\mathbf{p}$ | $\mathbf{q}$ | $\neg \mathbf{p}$ | $\neg \mathbf{q}$ | $\neg \mathbf{p} \wedge \neg \mathbf{q}$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | F |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

2. $(\neg p \wedge q) \vee \neg q$

Remember from Section 9.4... Disjunctions $(\mathrm{V})$ are true as long as at least one statement is true)

| $\mathbf{p}$ | $\mathbf{q}$ | $\neg \mathbf{p}$ | $\neg \mathbf{p} \wedge \boldsymbol{q}$ | $\neg \mathbf{q}$ | $(\neg \mathbf{p} \wedge \mathbf{q}) \vee \neg \mathbf{q}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | F | F |
| T | F | F | F | T | T |
| F | T | T | T | F | T |
| F | F | T | F | T | T |

3. $(p \Rightarrow q) \Leftrightarrow(p \wedge \neg q)$

According to Section 9.2 (and our formula sheet) If... then $(\Rightarrow)$ is only false when TF If and only if $(\Leftrightarrow)$ is only true when the two statements have the same truth value.

| $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{p} \Rightarrow \mathbf{q}$ | $\neg \mathbf{q}$ | $\mathbf{p} \wedge \neg \mathbf{q}$ | $(\mathbf{p} \Rightarrow \mathbf{q}) \Leftrightarrow(\mathbf{p} \wedge \neg \mathbf{q})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | F | F | F |
| T | F | F | T | T | F |
| F | T | T | F | F | F |
| F | F | T | T | F | F |

4. $(\neg q \vee \neg r) \wedge(\neg p \vee \neg q)$

| $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{r}$ | $\neg \mathbf{q}$ | $\neg \mathbf{r}$ | $\neg \mathbf{q} \vee \neg \mathbf{r}$ | $\neg \mathbf{p}$ | $\neg \mathbf{p} \vee \neg \mathbf{q}$ | $(\neg \mathbf{q} \vee \neg \mathbf{r}) \wedge(\neg \mathbf{p} \vee \neg \mathbf{q})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | F | F | F | F | F | F |
| T | T | F | F | T | T | F | F | F |
| T | F | T | T | F | T | F | T | T |
| T | F | F | T | T | T | F | T | T |
| F | T | T | F | F | F | T | T | F |
| F | T | F | F | T | T | T | T | T |
| F | F | T | T | F | T | T | T | T |
| F | F | F | T | T | T | T | T | T |

5. $(p \Rightarrow q) \Leftrightarrow(p \wedge \neg q)$

According to Section 9.2 (and our formula sheet) If... then $(\Rightarrow)$ is only false when TF If and only if $(\Leftrightarrow)$ is only true when the two statements have the same truth value.

| $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{p} \Rightarrow \mathbf{q}$ | $\neg \mathbf{q}$ | $\mathbf{p} \wedge \neg \mathbf{q}$ | $(\mathbf{p} \Rightarrow \mathbf{q}) \Leftrightarrow(\mathbf{p} \wedge \neg \mathbf{q})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | F | F | F |
| T | F | F | T | T | F |
| F | T | T | F | F | F |
| F | F | T | T | F | F |

6. $(\mathrm{p} \underline{\mathrm{V}} \mathrm{r}) \wedge \neg \mathrm{q}$

According to Section 9.4 (and our truth table) an exclusive disjunction $(\underline{\mathrm{V}})$ is true as long as only one statement is true.

| $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{r}$ | $\mathbf{p} \underline{\vee} \mathbf{r}$ | $\neg \mathbf{q}$ | $(\mathbf{p} \underline{\vee} \mathbf{r}) \wedge \neg \mathbf{q}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | F | F | F |
| T | T | F | T | F | F |
| T | F | T | F | T | F |
| T | F | F | T | T | T |
| F | T | T | T | F | F |
| F | T | F | F | F | F |
| F | F | T | T | T | T |
| F | F | F | F | T | F |

