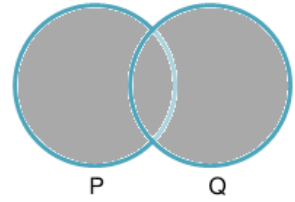


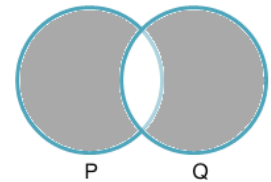
Section 9.4 Disjunction and Exclusive Disjunction

Definition: Disjunction ~ A disjunction is a statement formed by putting two propositions together such that the new statement is true whenever either or both sub-statements are true, but false when both are false. This is similar to saying $P \cup Q$. In symbols the disjunction of P and Q is denoted as $P \vee Q$



Example: If p is “It is at least 35° Celsius today” and q is “It is Friday” then $P \vee Q$ would be “It is at least 35° Celsius today or it is Friday or both”

Definition: Exclusive Disjunction ~ An exclusive disjunction is a disjunction where not both of the statement sub-statements can be true at once. It is therefore true when either proposition is true and it is false otherwise. This is similar to saying $P \cup Q - P \cap Q$. In symbols the disjunction of P and Q is denoted as $P \underline{\vee} Q$



Example: If p is “The penalty for being found guilty of murder is death” and q is “The penalty for being found guilty of murder is life imprisonment” then the exclusive disjunction is “Either the penalty for being found guilty of murder is death or life imprisonment”

Note: They may not necessarily use the word either. If the statement said “The penalty for being found guilty of murder is death or life imprisonment” and you weren’t sure if it was an exclusive “or” think about adding the statement “but not both” to the end. If that makes sense then it is an exclusive “or”

Another note: An inclusive “or” would be something like “It is raining or it is Friday”. It can be both.

Example: Determine the truth value of the given statements, given that P is true and Q is true.

a) $P \vee Q$

b) $P \vee \neg Q$

c) $\neg P \underline{\vee} \neg Q$

Answer: $T \vee T = T$

Answer: $T \vee F = T$

Answer: $F \underline{\vee} F = F$

Example: Determine the truth value of the given statements, given that P is true and Q is false.

a) $P \vee \neg Q$

b) $\neg P \underline{\vee} \neg Q$

c) $P \underline{\vee} \neg Q$

Answer: $T \vee T = T$

Answer: $F \underline{\vee} T = T$

Answer: $T \underline{\vee} T = F$

Example: Write each statement below in words.

P: It is sunny.

Q: It is cloudy.

a) $\neg P \vee Q$

Answer: Either it is not sunny or it is cloudy

b) $\neg P \wedge Q$

Answer: It is not sunny and it is cloudy.

c) $\neg P \underline{\vee} \neg Q$

Answer: Either it is not sunny or it is not cloudy (but not both)

Example: Write each of the following in symbolic form.

P: Simon is good at Mathematics.

Q: Simon does homework regularly.

R: Simon has passed his Mathematics test.

a) Simon does his homework regularly and he is good at Mathematics.

Answer: $Q \wedge P$

b) Simon does not do homework regularly and has failed his Mathematics Test

Answer: $\neg Q \wedge \neg R$

c) Either Simon is not good at Mathematics or he does not do homework regularly.

Answer: $\neg P \vee \neg Q$