Section 9.2 Implication and Equivalence

Definition: Implication ~ When proposition P being true means that another proposition Q must be true as well, we say that "P implies Q". We can then write "If P, then Q" In symbolic form this is $P \Longrightarrow Q$

The first proposition in an implication statement is called the antecedent of the statement.
The second proposition in an implication statement is called the consequent of the statement.

An implication is false when the antecedent is true but the consequent is false. That's because by definition if the first part of the statement is true the second part must also be true.

Definition: Equivalence Statement ~ If each statement P and Q imply each other we say that "If and only if P then Q". In symbolic form this is $P \Longleftrightarrow Q$

Therefore, if P is true Q is true. If P is false Q must also be false. If one proposition is true and the other is false then $P \Longleftrightarrow Q$ is a false statement.

Example: Determine the truth value of "If it rains on Saturday, then the Highlanders will win." given the following:

a) It *is* raining on Saturday and the Highlanders do win.

   This is consistent with the implicative statement. P is true. Q is true. So true.

b) It *is* raining on Saturday, but the Highlanders don't win.

   This is inconsistent with the implicative statement. P is true. Q is false. So false.

c) It is sunny on Saturday and the Highlanders win.

   This is consistent with the implicative statement, as no claim has been made regarding the outcome if it doesn't rain. P is not true. Q is true. So true.

d) It is sunny on Saturday and the Highlanders don't win.

   This is consistent with the implicative statement, as no claim has been made regarding the outcome if it doesn't rain. P is not true. Q is not true. So true.

Example: Consider the following logical propositions:
P: Isabelle is near-sighted.
Q: Isabelle can't see far away.

Write each of the following in words

a) $P \Longrightarrow Q$

If Isabelle is near-sighted then she can't see far away.

b) $Q \Longrightarrow P$

If Isabelle can't see far away then she is near-sighted.

c) $P \Leftrightarrow Q$

If and only if Isabelle is near-sighted then she can't see far away.

d) $\text{Not } P \Leftrightarrow \text{not } Q$

If and only if Isabelle isn't near-sighted then she can see far away.

Example: Write each of the following in symbolic logic form.

a) If it is sunny, then you must wear sunscreen.
b) If you must wear sunscreen, then it is sunny.
c) If and only if it is sunny then you must wear sunscreen.

Let P be "It is sunny" and let Q be "You must wear sunscreen"

Answers:

a) $P \Longrightarrow Q$

b) $Q \Longrightarrow P$

c) $P \Leftrightarrow Q$