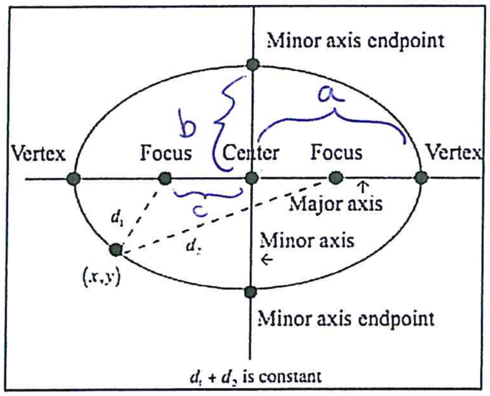
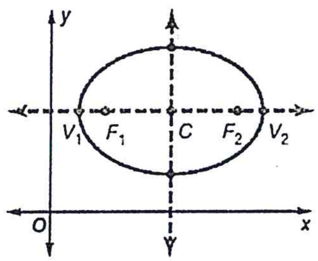
NOTES Section 9.1 Ellipses

An ellipse is a set of points such that the sum of the distances from a point P on the ellipse to the foci is constant. For ellipses, $0 < b < a$.



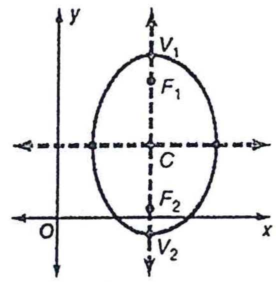
Standard Forms of Equations for Ellipses

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



Orientation: horizontal major axis
Center: (h, k)
Foci: $(h \pm c, k)$
Vertices: $(h \pm a, k)$
Co-vertices: $(h, k \pm b)$
Major axis: $y = k$
Minor axis: $x = h$
a, b, c relationship: $c^2 = a^2 - b^2$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$



Orientation: vertical major axis
Center: (h, k)
Foci: $(h, k \pm c)$
Vertices: $(h, k \pm a)$
Co-vertices: $(h \pm b, k)$
Major axis: $x = h$
Minor axis: $y = k$
a, b, c relationship: $c^2 = a^2 - b^2$

Length:
Major Axis $2a$
Minor Axis $2b$
Distance:
Center to a Foci c
Vertex to Center a
Co-Vertex to Center b

*****The placement of the bigger number (a) determines its orientation. If it's under the x^2 then it's horizontal, if it's under the y^2 then it's vertical. Also, the denominator of the x determines the x-intercepts and same with the y.**

*****If the a and b are the same value, then the ellipse is actually a circle.**

The eccentricity(e) is the measure of the "ovalness" of an ellipse. It is found by $e = \frac{c}{a}$ The closer it is to zero the more the ellipse looks like a circle. The closer it is to one the flatter the ellipse is (more oval like shape).

Example 1: Find the standard form of the equation of the ellipse centered at the origin with $\underbrace{\text{major axis of length 10}}_{2a}$ and $\underbrace{\text{foci at } (\pm 3, 0)}_{\text{on x-axis}}$

$(h,k) = (0,0)$

$2a$ on x-axis   so a^2 goes under x^2

$a = 5$ $c = 3$

$c^2 = a^2 - b^2$

$3^2 = 5^2 - b^2$

$-16 = -b^2$

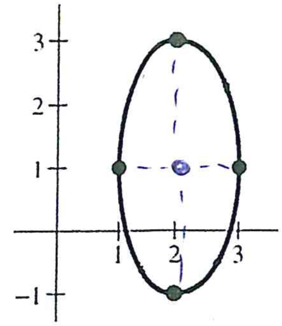
$b^2 = 16$ $b = 4$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-0)^2}{5^2} + \frac{(y-0)^2}{4^2} = 1$$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$

Example 2: Find the equation of the ellipse to the right.



center $(2,1)$
h,k

vertical (larger that way)
so a^2 goes under y^2

$2a=4$
$a=2$

$2b=2$
$b=1$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\frac{(x-2)^2}{1^2}+\frac{(y-1)^2}{2^2}=1$$

$$\boxed{\frac{(x-2)^2}{1}+\frac{(y-1)^2}{4}=1}$$

Example 3: Identify the conic as a circle or an ellipse. Then find the center, radius, vertices, foci, and eccentricity (as applicable), and sketch the graph.

a) $4x^2+9y^2=36$ $\div 36$

$$\frac{4x^2}{36}+\frac{9y^2}{36}=\frac{36}{36}$$

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

not a circle since $a\neq b$

center $(0,0)$

$a^2=9$ (since it's biggest)
$\leftrightarrow$ $a=3$

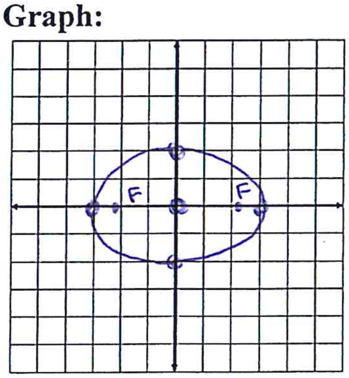
$b^2=4$
$\updownarrow$ $b=2$

$c^2=a^2-b^2$
$c^2=9-4$
$c=\sqrt{5}\approx 2.2$ $\leftrightarrow$

ecc: $e=\frac{c}{a}$

$$\boxed{e=\frac{\sqrt{5}}{3}}$$

Graph:



center
$(0,0)$

vertices
$(\pm 3,0)$

co-vertices
$(0,\pm 2)$

foci
$(\pm\sqrt{5},0)$

b) $25x^2+9y^2-200x+36y+211=0$

$25x^2-200x+9y^2+36y=-211$

$25(x^2-8x\quad)+9(y^2+4y\quad)=-211$

$(-8/2)^2=16$ $\quad$ $(4/2)^2=4$ $\quad$ $+25(16)+9(4)$

$25(x^2-8x+16)+9(y^2+4y+4)=-211+400+36$

$$\frac{25(x-4)^2+9(y+2)^2=225}{225}$$

$$\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$$

$a\neq b$
ellipse

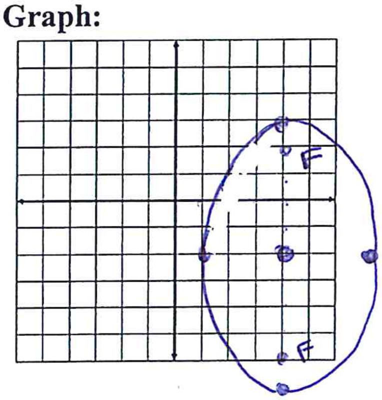
$a^2=25$
$a=5$ $\updownarrow$
$b^2=9$
$b=3$ $\leftrightarrow$

$c^2=a^2-b^2$
$c^2=25-9$
$c^2=16$
$c=4$ $\updownarrow$

ecc: $e=\frac{c}{a}$

$$\boxed{e=\frac{4}{5}}$$

Graph:



center
$(4,-2)$

vertices
$(4,3)(4,-7)$

co-vertices
$(1,-2)(7,-2)$

foci
$(4,2)(4,-6)$