7.7 More on extrema and optimization problems (Day 1)

SECOND DERIVATIVE TEST

If $f'(c) = 0$ and the second derivative of f exists near c, then

- If $f''(c) > 0$, then f has a relative minimum at $x = c$
- If $f''(c) < 0$, then f has a relative maximum at $x = c$
- If $f''(c) = 0$, the second derivative test fails and the first derivative test must be used to locate the relative extrema.

You do not HAVE to use the second derivative test, but it may save you time

Example 27: Find the relative extreme points of each function. Use the second derivative test whenever possible.

a.) $f(x) = x^3 - 3x^2 - 2$

$f'(x) = 3x^2 - 6x$

$3x(x-2) = 0$

$x = 0, 2$

or: max, min; $+$ $-$ $+$; 0 2 (plug #'s into $f'(x)$)

$f''(x) = 6x - 6$

$f''(0) = -$ max $\quad f(0) = (0,-2)$ max

$f''(2) = +$ min $\quad f(2) = (2,-6)$ min

b.) $f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$15x^2(x^2-1)$

$x = 0, 1, -1$

or: max, min; $+$ $-$ $-$ $+$; -1 0 1 (plug #'s into $f'(x)$)

$f''(x) = 60x^3 - 30x$

$f''(-1) = -$ max $\quad f(-1) = (-1,2)$ max

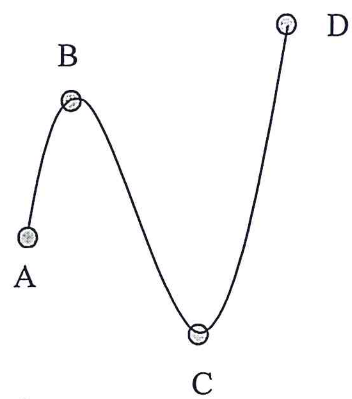
$f''(0) = 0 \rightarrow$ fails $\quad f(1) = (1,-2)$ min

$f''(1) = +$ min

Hmwk 7V

Definition: The <u>absolute (or global) extrema</u> of a function occur at either the relative extrema **or the endpoints of a function**.

Example 28:



a.) Identify each labeled point as an absolute maximum or minimum, a relative max or min, or neither.

abs. max = D $\quad$ relative max = B

abs min + relative min = C $\quad$ neither = A

b.) Find the absolute max and min for $f(x) = x^2 - 2x$ on $-1 \leq x \leq 2$.

$f'(x) = 2x - 2$

$2(x-1) = 0$

$x = 1$

$-$ $+$; 1

check at endpoints -1 and 2 and where $f'(x) = 0$

$f(-1) = 3 \leftarrow$ max #

$f(1) = -1 \leftarrow$ min #

$f(2) = 0$

abs max = 3

abs min = -1

Hmwk 7W

Include 7.7 day 1 notes with 7.6 day 2 notes

7.7 More on extrema and optimization problems (Day 2)

Definition: Word problems involve finding the maximum or minimum values such as maximizing area or minimizing cost are known as **optimization problems**.

Tips for solving optimization problems:

1. DRAW A PICTURE!
2. Assign your variables
3. Write an equation to be **optimized** in terms of two variables
4. Find values that are *sensible* for the problem where the derivative equals zero
5. Verify that you have the desired max or min using the first or second derivative test.

***Remember to check the endpoints on a closed interval!**

Example 29: The product of two positive numbers is 48. Find the two numbers so that the sum of the first number plus three times the second number is a minimum.

$x =$ 1st pos #

$y =$ 2nd pos #

$xy = 48 \rightarrow y = 48/x$

$x + 3y =$ Something

$x + 3(48/x) = S$

$x + \frac{144}{x} = S$

$x + 144x^{-1} = S$

$1 - 144x^{-2} = S'(x)$

$1 - \frac{144}{x^2} = 0$

$\frac{144}{x^2} = 1$

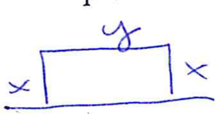
$x^2 = 144 \quad x = \pm 12$

plug in $x=12$ (can't be negative)

$y = 4$

$\boxed{\text{Answer } 12 \text{ \& } 4}$

Example 30: A rectangular plot of farmland is enclosed by 180 m or fencing material on three sides. The fourth side of the plot is bounded by a stone wall. Find the dimensions of the plot that enclose the maximum area. Find the maximum area.



$A = xy$

$180 = 2x + y$

$180 - 2x = y$

$A = (180 - 2x)x$

$= 180x - 2x^2$

$A' = 180 - 4x$

$0 = 180 - 4x \quad x = 45\text{m}$

$y = 90\text{m}$

Area $=$

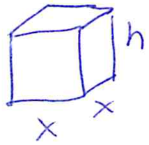
$45 \cdot 90 =$

4050 m^2

Hmwk 7X

Include 7.7 day 1 notes with 7.6 day 2 notes

Example 31: Find the dimensions of an open box with a square base and surface area of 192 square centimeters that has a maximum volume.



$SA = 192 cm^2$

$SA = x^2 + 4xh$

$192 = x^2 + 4xh$

$\frac{192 - x^2}{4x} = h$

$V = x^2 h$

$V = x^2 \left(\frac{192 - x^2}{4x} \right)$

$V = \frac{192x - x^3}{4}$

$V = 48x - \frac{1}{4}x^3$

$V' = 48 - 3/4 x^2$

$0 = 48 - 3/4 x^2$

$x^2 = 64 \quad x = \pm 8cm$

$x = 8 cm$

$h = 4 cm$

$\boxed{8 \times 8 \times 4 cm}$

Example 32: The cost C of ordering and storing x units of a product is $C(x) = x + \frac{10,000}{x}$. A delivery truck can deliver at most 200 units per order. Find the order size that will minimize the cost.

$C(x) = x + 10,000x^{-1}$

$C'(x) = 1 - 10,000x^{-2}$

$0 = 1 - \frac{10,000}{x^2}$

$x^2 = 10,000$

$x = \pm 100$ units

$x = 100$ units

Try min, max & where $C'(x) = 0$

$C(1) = 10,001$

$C(100) = \$200 \leftarrow$ minimum #

$C(200) = 250$

100 units = minimum cost

Hmwk 74

Include 7.7 day 1 notes with 7.6 day 2 notes