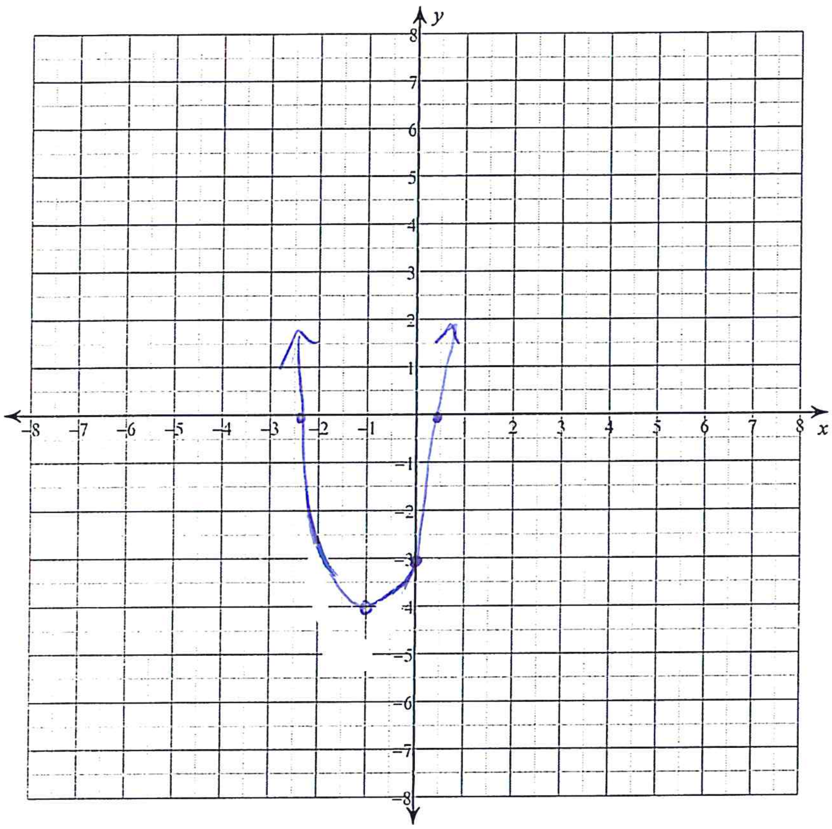
IB Math SL   Name Key   ID: 1

Curve Sketching Worksheet

For each problem, find the: x and y intercepts, asymptotes (if applicable), x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = 2x^2 + 4x - 2$



intercepts:

$y = 2(0)^2 + 4(0) - 2$ $\boxed{y = -2}$

$0 = 2x^2 + 4x - 2$

$0 = 2(x^2 + 2x - 1)$

$\frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$ $x \approx -1 \pm 1.4$

$\boxed{x \approx 0.4, -2.4}$

min/max:

$y' = 4x + 4$

$0 = 4x + 4$

$x = -1$

y': $-$ (decreasing) | $+$ (increasing) at -1

min $f(-1) = -$ $\boxed{\text{min } (-1, -4)}$

$\boxed{\text{decrease: } (-\infty, -1) \quad \text{increase } (-1, \infty)}$

concavity:

$y'' = 4$

since y'' is pos all the time

CC↑ all the time

$\boxed{CC\uparrow \ (-\infty, \infty)}$

x-coord. critical points:

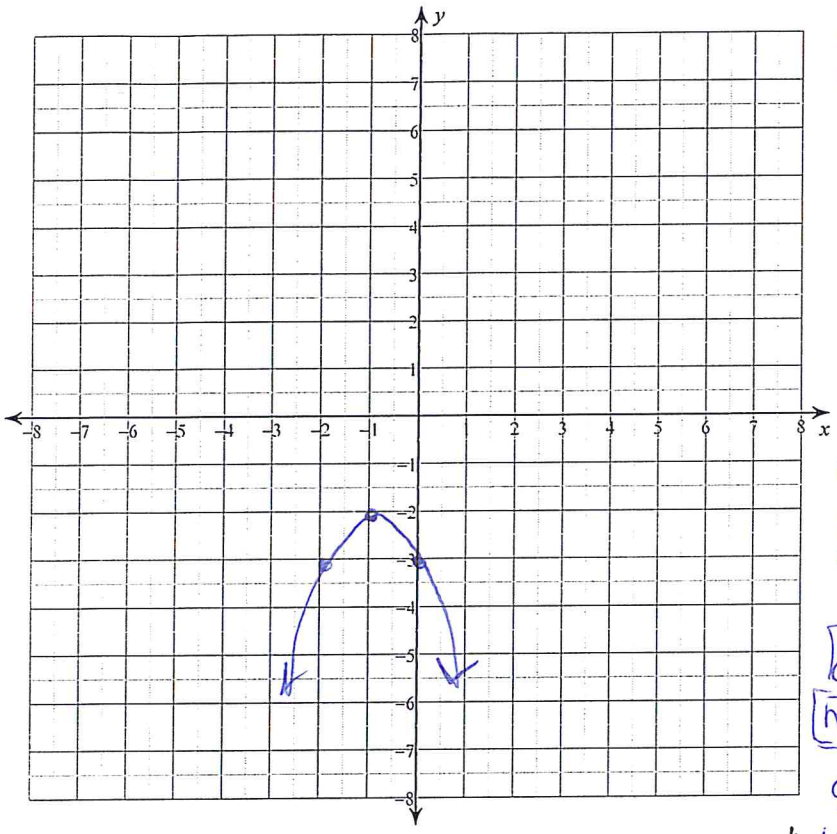
(where $y' = 0$ or undefined)

$\boxed{x = -1}$

no asymp.

no inflection points

2) $y = -x^2 - 2x - 3$



intercepts:

$y = -(0)^2 - 2(0) - 3$ $\quad y = -3$

$0 = -x^2 - 2x - 3$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(-3)}}{2(-1)}$

$x = \frac{2 \pm \sqrt{4-12}}{-2}$ → imaginary

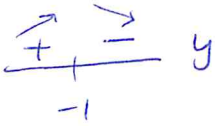
no x-int.

min/max:

$y' = -2x - 2$

$0 = -2x - 2$

$x = -1$ critical point



max $f(-1) = -2$

max $(-1, -2)$ inc $(-\infty, -1)$ dec $(-1, \infty)$

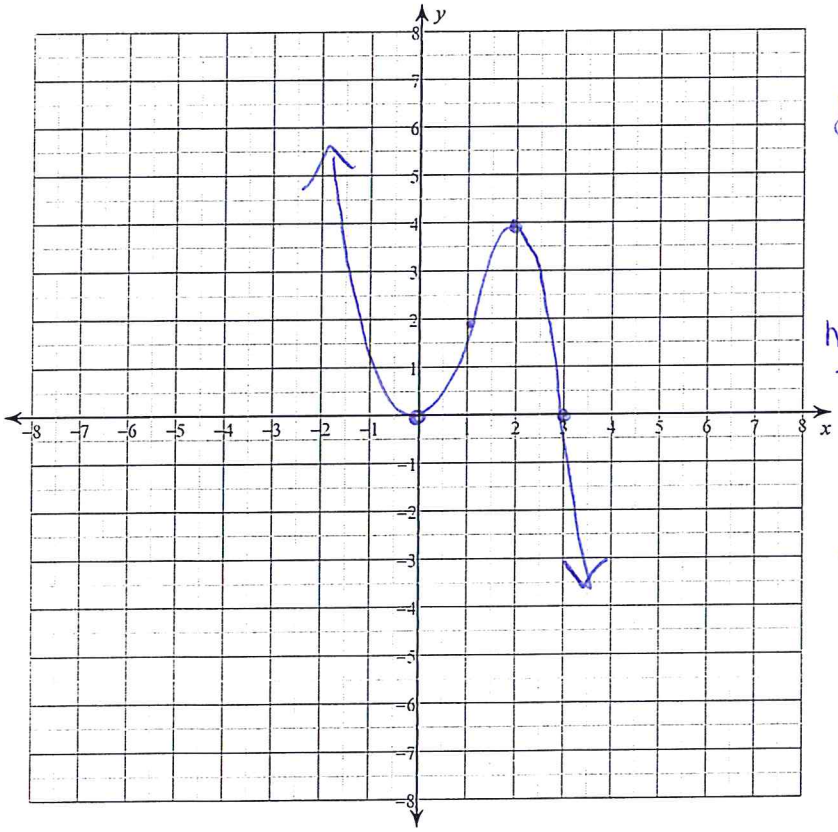
concavity:

$y'' = -2$

since y'' is neg all the time

CC↓ $(-\infty, \infty)$

3) $y = -x^3 + 3x^2$



intercepts:

$y = -(0)^3 + 3(0)^2$ $\quad y = 0$

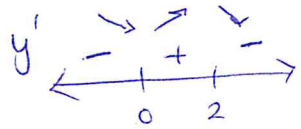
$0 = -x^3 + 3x^2$

$0 = -x^2(x - 3)$ $\quad x = 0, x = 0, x = 3$

even multiplicity

min/max:

$y' = -3x^2 + 6x$



$0 = -3x^2 + 6x$

$0 = -3x(x - 2)$

$x = 0$ $x = 2$ critical point

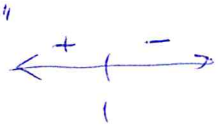
min $f(0) = 0$

max $f(2) = 4$

inc $(0, 2)$ dec $(-\infty, 0)(2, \infty)$

concavity:

$y'' = -6x + 6$



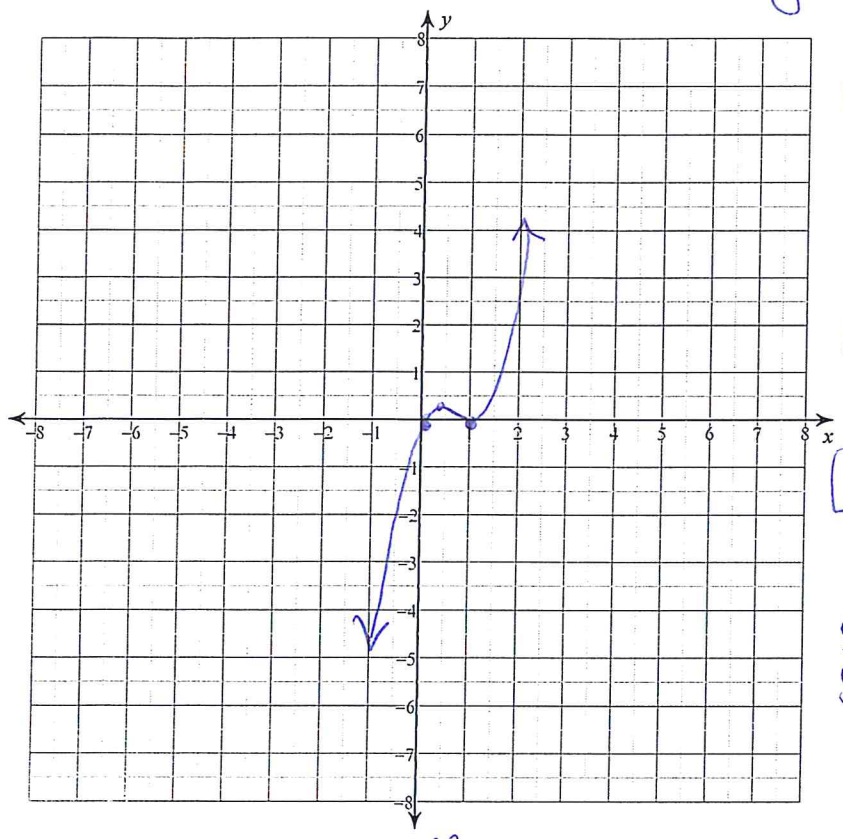
$0 = -6x + 6$

$x = 1$

inf pt $f(1) = 2$

CC↑ $(-\infty, 1)$; CC↓ $(1, \infty)$

4) $y = x^3 - 2x^2 + x$ * Scientific calc ally!



intercepts:

$y = 0^3 - 2(0)^2 + 0$ $\boxed{y=0}$

$0 = x(x^2-2x+1)$

$x(x-1)(x-1)$ $\boxed{x=0, x=1, x=1}$ even multiplicity

min/max:

$y' = 3x^2-4x+1$

$0 = (3x-1)(x-1)$

$\boxed{x = 1/3 \quad x=1}$ critical points

y' + (1/3) − (1) +

$\boxed{f(1/3) \approx .15 \text{ max} \quad f(1)=0 \text{ min}}$

$\boxed{\text{inc } (-\infty, 1/3)(1,\infty) \quad \text{dec } (1/3, 1)}$

concavity:

$y'' = 6x-4$

$0 = 6x-4$

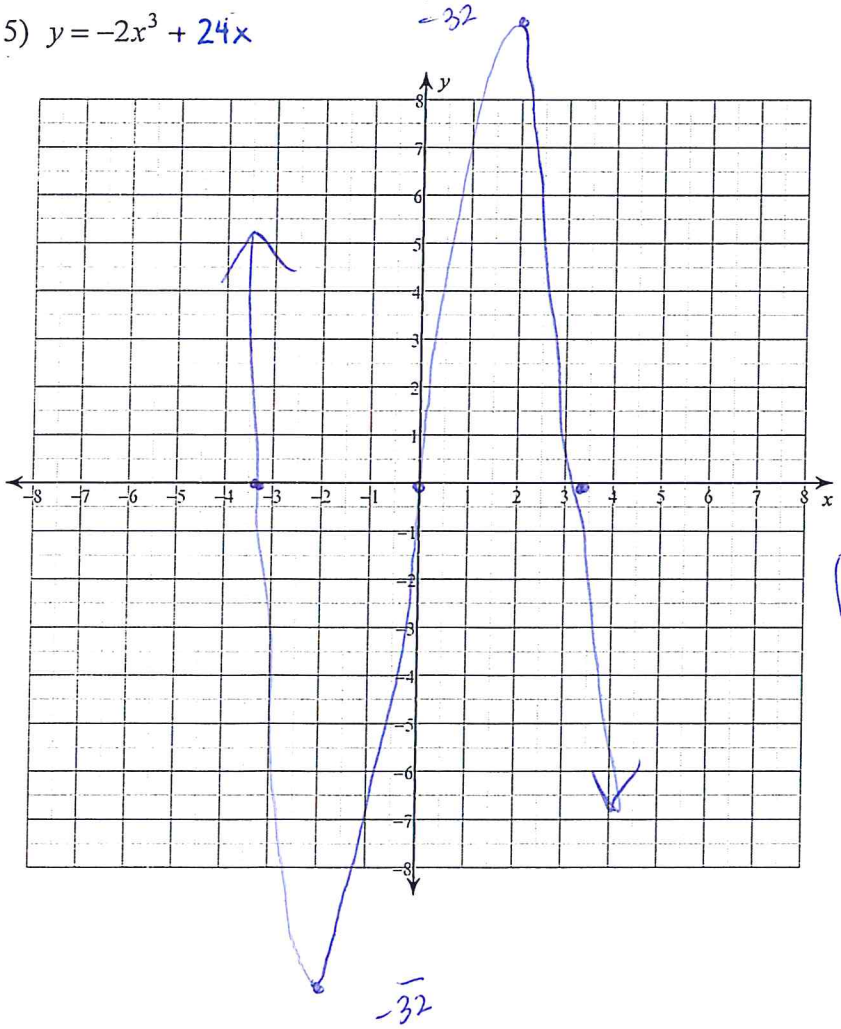
$x = 2/3$

y'' − (2/3) +

inf pt $(2/3, -.074)$

$\boxed{cc\downarrow (-\infty, 2/3) \quad cc\uparrow (2/3, \infty)}$

5) $y = -2x^3 + 24x$



intercepts:

$y = -2(0)^3 + 24(0)$ $\boxed{y=0}$

$0 = -2x^3 + 24x$

$0 = -2x(x^2-12)$

$\boxed{x = 0, x = \pm\sqrt{12} \approx \pm 3.}$

min/max:

$y' = -6x^2+24$

$0 = -6(x^2-4)$

$\boxed{x = \pm 2}$ critical points

y' − (−2) + (2) −

$\boxed{\text{min } f(-2) = -32 \quad \text{max } f(2) = 32}$

$\boxed{\text{dec } (-\infty,-2)(2,\infty) \quad \text{inc } (-2,2)}$

concavity:

$y'' = -12x$

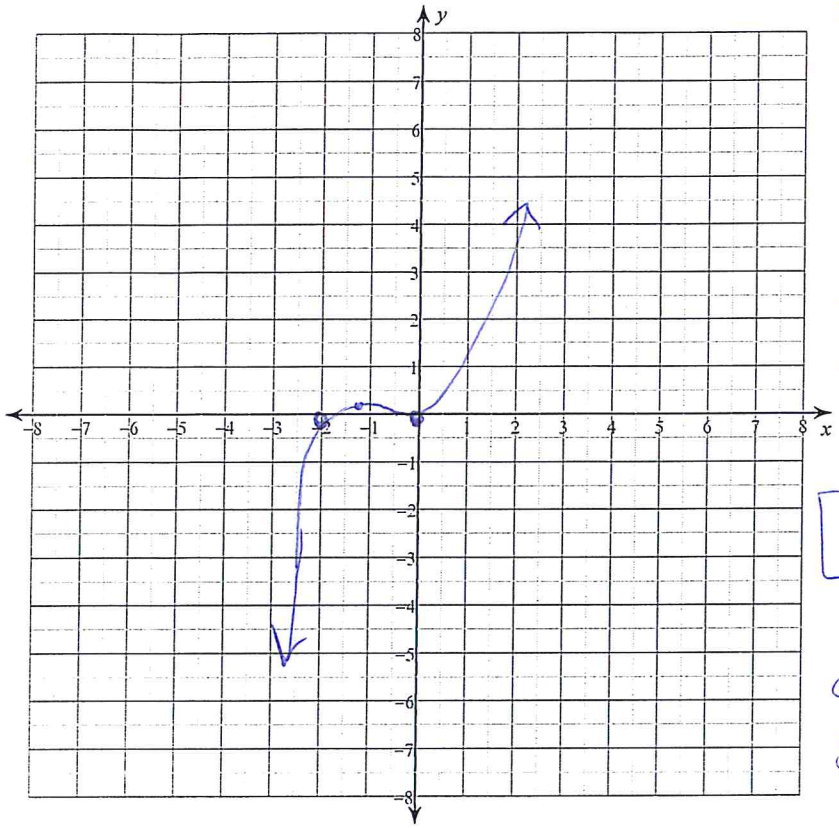
$x = 0$

y'' + (0) −

$\boxed{\text{inf. pt } f(0) = 0}$

$\boxed{cc\uparrow (-\infty, 0) \quad cc\downarrow (0, \infty)}$

6) $y = \frac{x^3}{3} + \frac{2x^2}{3}$ $y = \frac{1}{3}x^3 + \frac{2}{3}x^2$



intercepts:

$y = \frac{0^3}{3} + \frac{2}{3}(0)^2$ $y = 0$

$0 = \frac{1}{3}x^2(x+2)$

$x = 0, x = 0, x = -2$

even multiplicity

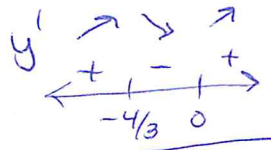
min/max:

$y' = x^2 + 4/3x$

$0 = x(x+4/3)$

$x = 0, -4/3$

critical points



$f(0) = 0$ $f(-4/3) \approx 0.4$

min max

inc $(-\infty, -4/3)(0, \infty)$

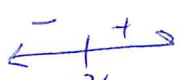
dec $(-4/3, 0)$

concavity:

$y'' = 2x + 4/3$

$0 = 2x + 4/3$

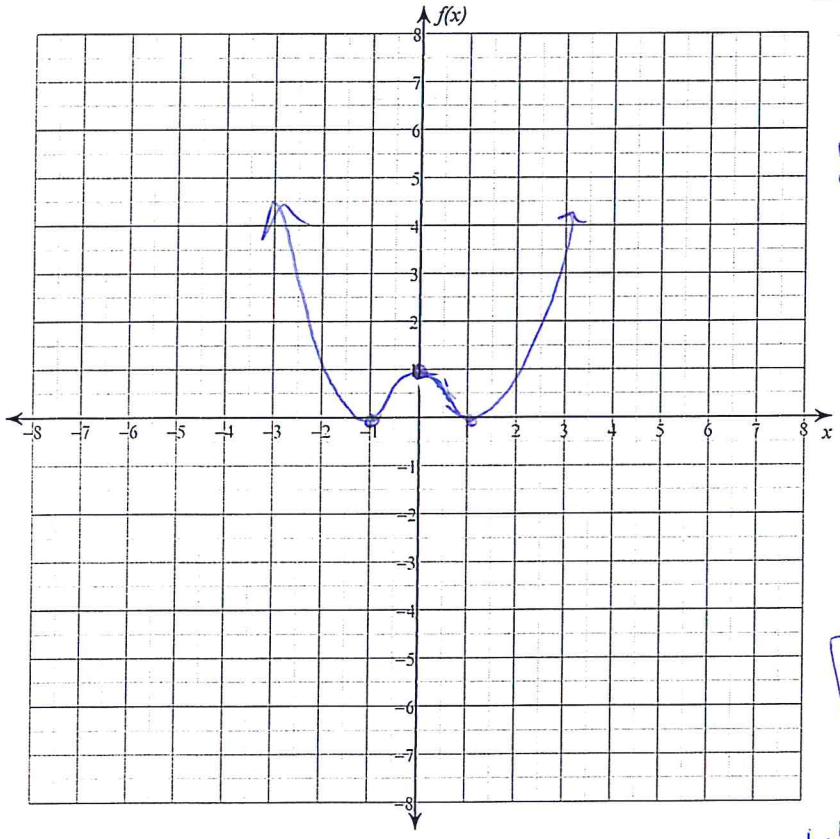
$x = -2/3$



inf. pt $f(-2/3) \approx .25$

CC ↓ $(-\infty, -2/3)$ CC ↑ $(-2/3, \infty)$

7) $f(x) = x^4 - 2x^2 + 1$



intercepts:

$y = 0^4 - 2(0)^2 + 1$ $y = 1$

$0 = x^4 - 2x^2 + 1$

$0 = (x^2 - 1)(x^2 - 1)$

$x = -1, -1, 1, 1$

even multiplicity

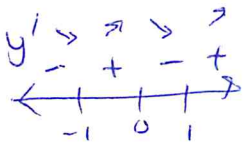
min/max:

$y' = 4x^3 - 4x$

$0 = 4x(x^2 - 1)$

$x = 0, 1, -1$

critical points



$f(-1) = 0$ $f(0) = 1$ $f(1) = 0$

min max min

inc: $(-1, 0)(1, \infty)$

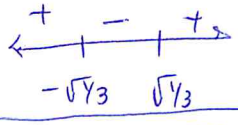
dec: $(-\infty, -1)(0, 1)$

concavity:

$y'' = 12x^2 - 4$

$0 = 4(3x^2 - 1)$

$x = \pm\sqrt{1/3}$

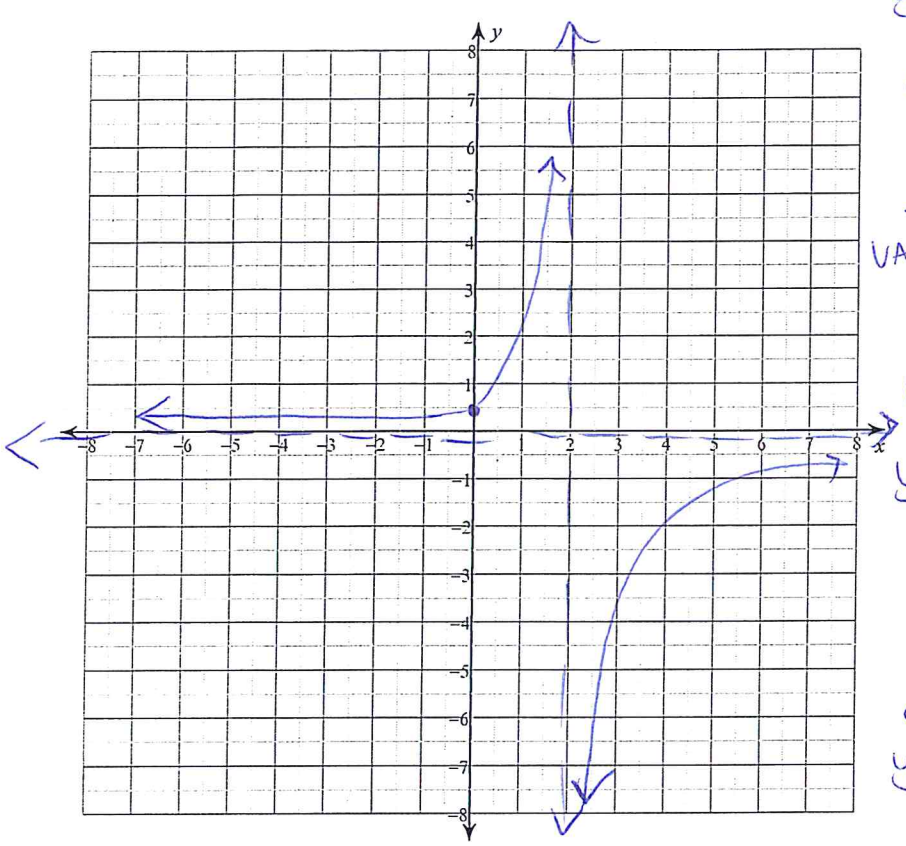


inf pt $(\pm\sqrt{1/3}, 4/9)$

CC ↑ $(-\infty, -\sqrt{1/3})$ $(\sqrt{1/3}, \infty)$

CC ↓ $(-\sqrt{1/3}, \sqrt{1/3})$

8) $y = -\dfrac{1}{x-2}$



intercepts:

$y = \frac{-1}{0-2} \Rightarrow y = 1/2$

$0 = \frac{-1}{x-2}$ $0 = -1$ no x-int.

Asym

VA: $x-2=0$ $x=2$ HA: $y=0$

critical pt $x=2$

min/max:

$y' = \frac{0(x-2)-1(-1)}{(x-2)^2} = \frac{1}{(x-2)^2}$

y': + | + (2)

$0 = \frac{1}{(x-2)^2}$ $0 = 1$ no min/max

inc $(-\infty, 2)(2, \infty)$

concavity:

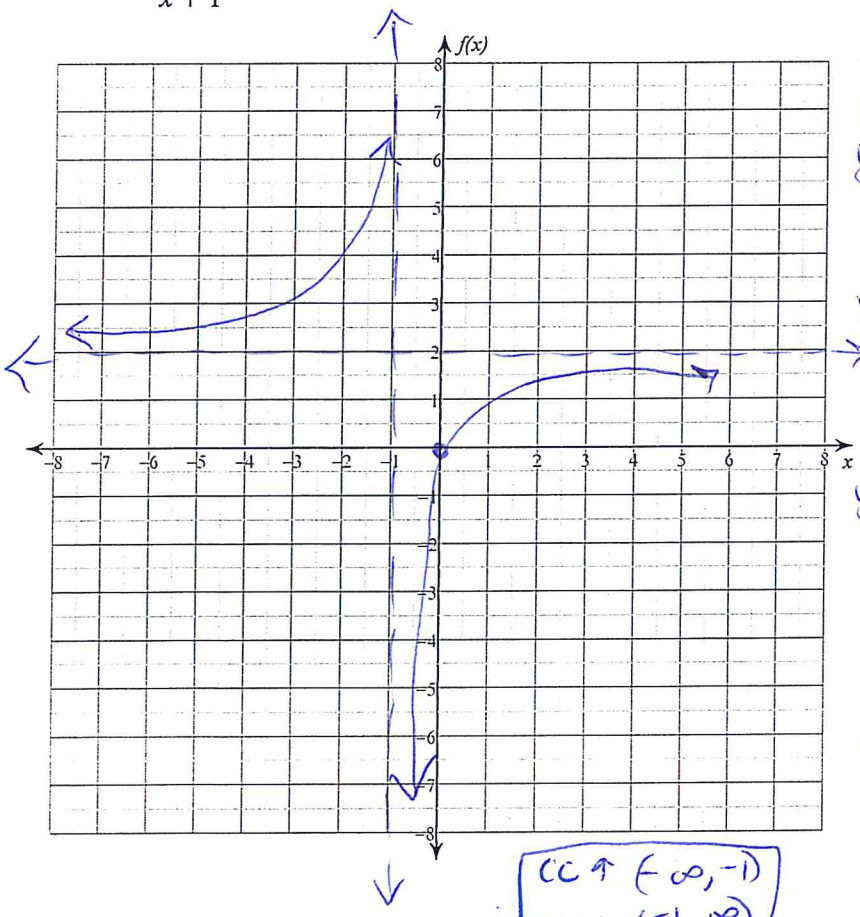
$y'' = \frac{0(x-2)^2 - 2(x-2)^1 \cdot 1(1)}{((x-2)^2)^2} = \frac{-2}{(x-2)^3}$

y'': + | − (2)

$0 = 2$ no infl pt

cc↑ $(-\infty, 2)$ cc↓ $(2, \infty)$

9) $f(x) = \dfrac{2x}{x+1}$



intercepts:

$y = \frac{2(0)}{0+1}$ $y = 0$

$0 = \frac{2x}{x+1}$ $0 = 2x$ $x = 0$

Asym

VA: $x+1=0$ $x=-1$ HA: $\frac{2x}{x}$ $y=2$

critical pt $x=-1$

min/max:

$y' = \frac{2(x+1)-1(2x)}{(x+1)^2} = \frac{2}{(x+1)^2}$

y': + | + (−1)

$0 = \frac{2}{(x+1)^2}$ $0 = 2$ no min/max

inc $(-\infty, -1)(-1, \infty)$

concavity:

$y'' = \frac{0(x+1)^2 - 2(x+1)^1 \cdot 1 \cdot 2}{((x+1)^2)^2} = \frac{-4}{(x+1)^3}$

y'': + | − (−1)

$0 = \frac{-4}{(x+1)^3}$ $0 = -4$ no infl pt

cc↑ $(-\infty, -1)$

cc↓ $(-1, \infty)$