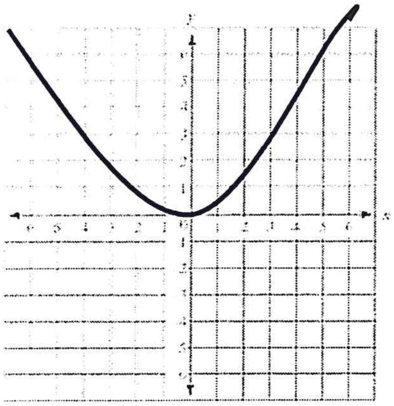
7.6 The Derivative and Graphing (2 days?)

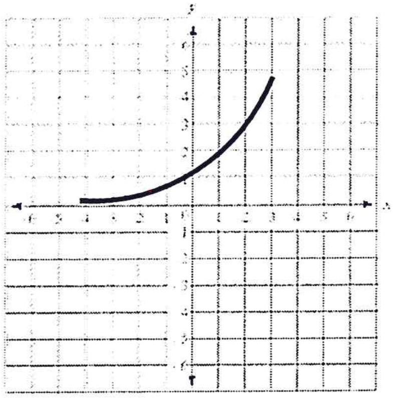
Definition: A function is **increasing** when the function has a positive slope.
Definition: Similarly, a function is **decreasing** when the function has a negative slope.

Example 21: Write the intervals on which the function is increasing and decreasing.

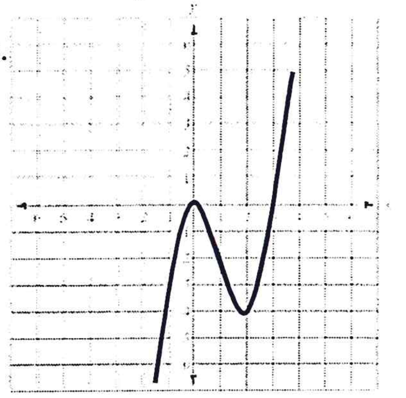
a.



b.



c.



a. Inc: $(0,\infty)$
Dec: $(-\infty,0)$

b. Inc: $(-\infty,\infty)$
Dec: none

c. Inc: $(-\infty,0)\cup(2,\infty)$
Dec: $(0,2)$

Definition: If $f'(x) > 0$ for all x in (a,b), then f is **increasing** on (a,b).

Definition: If $f'(x) < 0$ for all x in (a,b), then f is **decreasing** on (a,b).

Definition: A **stationary point** is where $f'(x) = 0$.

Definition: A **critical number** of f is a point where $f'(x) = 0$ OR $f'(x)$ is undefined.

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Example 22: Use the derivative of f to find the intervals on which f is increasing or decreasing.

a.) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

$6(x^2 - x - 2) = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

$+ \quad - \quad +$

$-1 \quad 2$

(plug #'s into f'(x))

increasing $(-\infty,-1)\cup(2,\infty)$
decreasing $(-1,2)$

b.) $f(x) = \frac{x^2-4}{x^2-1}$

$$\frac{2x(x^2-1) - 2x(x^2-4)}{(x^2-1)^2}$$

$$\frac{2x^3 - 2x - 2x^3 + 8x}{(x^2-1)^2}$$

$$\frac{6x}{(x^2-1)^2} = 0$$

$6x = 0 \quad x = 0$
undefined @ ±1

$- \quad - \quad + \quad +$

$-1 \quad 0 \quad 1$

increasing $(0,1)\cup(1,\infty)$
decreasing $(-\infty,-1)\cup(-1,0)$

c.) $f(x) = x^3$

$f'(x) = 3x^2$

$3x^2 = 0$

$x = 0$

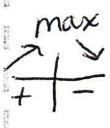
$+ \quad +$

0

increasing: $(-\infty,0)\cup(0,\infty)$
decreasing: none

FIRST DERIVATIVE TEST

Definition: A function has a **relative maximum** (or local) when the function changes from increasing to decreasing. Hence, if $f'(x)$ changes from positive to negative at x = c, then f has a relative maximum at (c, f(c)).

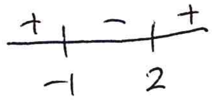


Definition: A function has a **relative minimum** (or local) when the function changes from decreasing to increasing. Hence, if f' (x) changes from negative to positive at x = c, then f has a relative minimum at (c, f(c)).

***Note:** if $f'(x)$ does not have to change sign at a critical number. If this happens, there is neither a relative maximum or minimum at that point.

Example 23: Use the first derivative test to find the relative extrema for the functions in Example 22. (copy diagrams from last example)

a.) $f(x) = 2x^3 - 3x^2 - 12x$



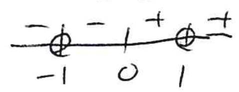
max@ x=−1

min@ x=2

f(−1)=7 f(2)=−20

relative maximum(−1,7) relative minimum(2,−20)

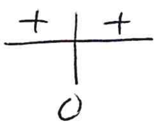
b.) $f(x) = \frac{x^2-4}{x^2-1}$



min@x=0 max: none

relative min (0,4)

c.) $f(x) = x^3$



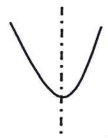
no max

no min

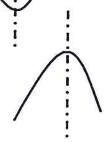
Hmwk 7R

Concavity TEST

Definition: If $f''(x) > 0$ for all x in (a,b) then f is concave up on (a,b).



Definition: If $f''(x) < 0$ for all x in (a,b), then f is concave down on (a,b).



Definition: A point on the graph of f is an **inflexion point** if $f''(x) = 0$ AND if f'' changes signs.

Example 24: For functions from Example 22, use the second derivative to find the intervals where the function is concave up and concave down. Also find the inflexion points. see bk pg 236

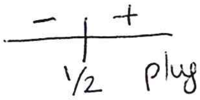
a.) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

$f''(x) = 12x - 6$

$12x - 6 = 0$

$x = 1/2$



plug into $f''(x)$

CC↑ $(1/2, \infty)$

CC↓ $(-\infty, 1/2)$

inf. pt $(1/2, f(1/2))$ $(1/2, -6.5)$

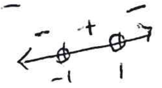
b.) $f(x) = \frac{x^2-4}{x^2-1}$ $f'(x) = \frac{6x}{(x^2-1)^2}$

$$f''(x) = \frac{6(x^2-1)^2 - 2(x^2-1)\cdot 2x \cdot 6x}{((x^2-1)^2)^2}$$

$$\frac{6(x^2-1)^2 - 24x^2(x^2-1)}{(x^2-1)^4}$$

$$\frac{(x^2-1)[6x^2-6-24x^2]}{(x^2-1)^4}$$

$$\frac{-18x^2-6}{(x^2-1)^3} = 0$$



$-18x^2 - 6 = 0$

$x^2 = -\frac{1}{3}$

no solution

$x \neq \pm 1$

no inflection

CC↑ $(-1, 1)$

CC↓ $(-\infty, -1)(1, \infty)$

c.) $f(x) = x^3$

Hmwk 7S

Hmwk 7T

X-int: $0 = 2x^3 - 3x^2 - 12x$

$x(2x^2-3x-12)$ → Quad. Formula!

$x=0$

Example 25: Sketch the graph of each function. Use the information you found in Examples 22-24 and intercepts and asymptotes to help draw the graph.

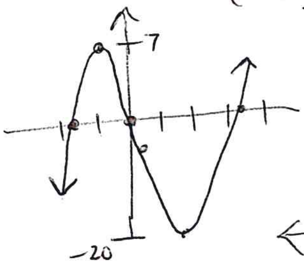
Vertical $x^2-1=0$ $x=\pm1$

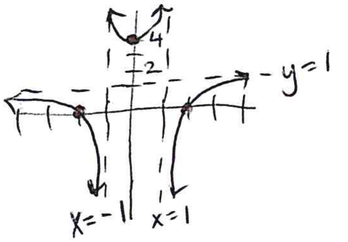
Hor. Asym. $\frac{1x^2}{1x^2}$ $y=1$

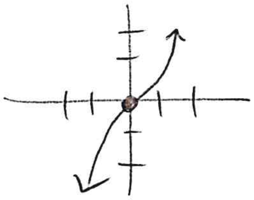
	a.) $f(x) = 2x^3 - 3x^2 - 12x$	b.) $f(x) = \frac{x^2-4}{x^2-1}$	c.) $f(x) = x^3$
inc:	$(-\infty,-1)\cup(2,\infty)$	$(0,1)\cup(1,\infty)$	$(-\infty,\infty)$
dec:	$(-1,2)$	$(-\infty,-1)\cup(-1,0)$	none
relative max:	$(-1,7)$	none	none
relative min:	$(2,-20)$	$(0,4)$	none
concave down:	$(-\infty,\frac{1}{2})$	$(-\infty,-1)(1,\infty)$	$(-\infty,0)$
concave up:	$(\frac{1}{2},\infty)$	$(-1,1)$	$(0,\infty)$
inflexion point:	$(\frac{1}{2},-6.5)$	none (asymp pt)	$(0,0)$
x-int:	$(0,0)$ $(\frac{3\pm\sqrt{108}}{4},0)$	$(2,0)(-2,0)$	$(0,0)$
y-int:		$(0,4)$	$(0,0)$

→ $(0,0)$

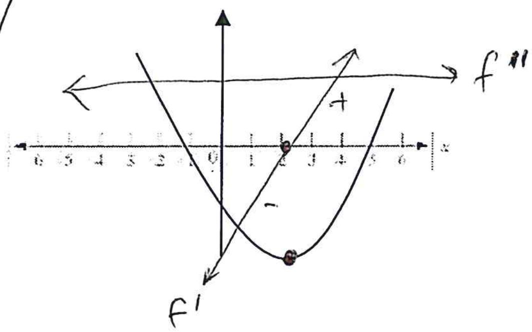
→ $(-1.8,0)$ $(3.31,0)$







Example 26:



Hmwk 7U

a.) Given the graph shown (pg. 239) is a graph of f, sketch the graphs of f', and f''.

f is dec. $(-\infty,2)$ → f' is neg $(-\infty,2)$

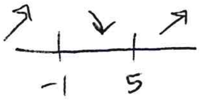
inc $(2,\infty)$ → f' is pos $(2,\infty)$

f' is zero at 2

Since f'(x) is linear f''(x) must be a constant

Since f is always concave up, it's a positive constant

b.) Given the graph shown is a graph of f', sketch graphs of f and f''.

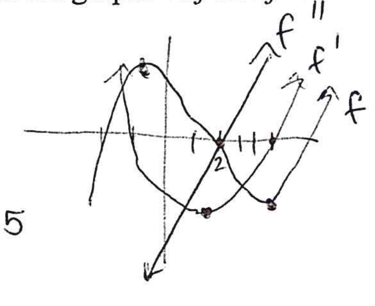


since f' is negative from (-1,5)

f is decreasing (-1,5)

since f' is positive $(-\infty,-1)$ and $(5,\infty)$

f is increasing + max@ x=-1, min x=5



f'' is linear because of discussion above