7.4 The Chain Rule and Higher Order Derivatives (Day 2)

Definition: The derivative $f'(x)$ or dy/dx is called the **first derivative** of y with respect to x.
Definition: The **second derivative** of y with respect to x can be written as $f''(x)$ (said f double prime) or d^2y/dx^2. (A way to remember the notation is $\frac{d}{dx}(\frac{dy}{dx})$).
Definition: The **third derivative** of y with respect to x is written as $f'''(x)$ or d^3y/dx^3.
Definition: **Higher derivatives** are second derivatives and higher.

*Note: Above third derivatives, you denotes as $f^n(x)$ where n is the number of derivative. For example, the forth derivative is denoted as $f^4(x)$.

*Note: Also, in physics, first derivative is **velocity**, second derivative is **acceleration**, and third derivative is **jerk**.

Example 15 (a): Find the first three derivatives of $f(x) = x^4 + 3x^2 + x$.

$f'(x) = 4x^3 + 6x + 1 \quad f''(x) = 12x^2 + 6 \quad f'''(x) = 24x$

Example 15 (b): If $f'(x) = \sqrt{x^2 + 4}$, find $f''(x)$. $\quad f'(x) = (x^2+4)^{\frac{1}{2}}$

$f''(x) = \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+4}}$

* **Example 15 (c):** If $y = 4e^{2x}$, find $\left.\frac{d^3y}{dx^3}\right|_{x=1}$

typo

$\frac{dy}{dx} = 4e^{2x} \cdot 2 = 8e^{2x}$

$\frac{d^2y}{dx^2} = 8e^{2x} \cdot 2 = 16e^{2x}$

$\frac{d^3y}{dx^3} = 32e^{2x}$

now plug in x=1 $\boxed{32e^2}$

* **Example 15 (d):** If $s(t) = -16t^2 + 16t + 32$, find $\cancel{\frac{d^2s}{ds^2}}$. $\frac{d^2s}{dt^2}$

typo

$\frac{ds}{dt} = -32t + 16$

$\frac{d^2s}{dt^2} = \boxed{-32}$

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