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7.4 The Chain Rule and Higher Order Derivatives (Day 1)

Finding the derivative of $3x^{10} + 2$ is fairly simple. What if you were to find $(3x + 2)^{10}$? Instead of F.O.I.L.ing that out or using Pascal's triangle, we have a rule in Calculus to help us.

IB Formula sheet

Chain Rule: If $f(x) = u(v(x))$, then $f'(x) = u'(v(x)) \cdot v'(x)$ $\quad y = g(u),\ u = f(x)\quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

→ Take the derivative of the **outside** function, times the derivative of the **inside**

Example 12: Each function is in the form $f(x) = u(v(x))$. Identify u(x) and v(x), then find the derivative of f.

a. $f(x) = 4(5x^3 + 2)^6$

(inside) $v(x) = 5x^3 + 2$

(outside) $u(x) = 4x^6$

$f'(x) = u'(v(x)) \cdot v'(x)$

$24(5x^3+2)^5 \cdot 15x^2$

$\boxed{360x^2(5x^3+2)^5}$

b. $f(x) = \sqrt{4x^2 + 1}$

$v(x) = 4x^2+1$

$u(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(4x^2+1)^{-\frac{1}{2}} \cdot 8x$

$\boxed{\dfrac{4x}{\sqrt{4x^2+1}}}$

c. $f(x) = e^{x^2}$

$v(x) = x^2$

$u(x) = e^x$

$f'(x) = e^{x^2} \cdot 2x$

$= \boxed{2xe^{x^2}}$

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Example 13: Use chain rule to find the derivative of $f(x) = \frac{1}{x^2+1}$. $= (x^2+1)^{-1}$

*Reminder: Always rewrite the function BEFORE taking the derivative, if possible.

$f'(x) = -1(x^2+1)^{-2} \cdot 2x = \dfrac{-2x}{(x^2+1)^2}$

Example 14: Find the derivative.

a. $f(x) = x\sqrt{1 - x^2} = x(1-x^2)^{\frac{1}{2}}$

Product rule!

$u'v + uv'$

$f'(x) = 1(1-x^2)^{\frac{1}{2}} + x\left[\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x\right]$

$\boxed{\sqrt{1-x^2} + \dfrac{-x^2}{\sqrt{1-x^2}}}$

b. $f(x) = e^{2(3x-1)^4}$

$f'(x) = e^{2(3x-1)^4} \cdot 8(3x-1)^3 \cdot 3$

$\boxed{24(3x-1)^3 e^{2(3x-1)^4}}$

c. $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

→ Quotient Rule!

$f'(x) = \dfrac{1}{\frac{x}{x^2+1}} \cdot \left(\dfrac{1(x^2+1) - 2x(x)}{(x^2+1)^2}\right)$

$\dfrac{\cancel{x^2+1}}{x}\left(\dfrac{x^2+1-2x^2}{(x^2+1)^{\cancel{2}}}\right)$

$\boxed{\dfrac{-x^2+1}{x(x^2+1)}}$

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