7.3 More Rules for Derivatives

More Derivative Rules

Derivative of e^x: If $f(x) = e^x$, then $f'(x) = e^x$

Derivative of ln x: If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$

Example 8: Find the derivative of each function.

a.) $f(x) = 3e^x$

$f'(x) = 3e^x$

b.) $f(x) = x^2 + \ln x$

$f'(x) = 2x + \frac{1}{x}$

c.) $f(x) = \ln e^{3x} = 3x \ln e = 3x$

$f'(x) = 3$

More Derivative Rules

Careful…when finding the derivative of a product or quotient, it's easy to make a mistake.

Let's try to find the derivative of $f(x) = x^2 \cdot x$. We would think $f'(x) = 2x \cdot 1 = 2x$. However, using algebra rules $f(x) = x^3$ by adding exponents. Meaning, $f'(x) = 3x^2$, not 2x!

Product Rule: If $f(x) = u(x) \cdot v(x)$, then $f'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$
Note: your book says it in reverse
→In calculus, we said f 'g +g'f.

IB formula sheet

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient Rule: If $f(x) = u(x) / v(x)$, then $f'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{[v(x)]^2}$

→ In calculus, we said (f 'g - g'f.) / g^2

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Example 9: Find the derivative of each function.

a.) $f(x) = \underset{u}{(3x + 1)}\underset{v}{(\ln x)}$

$u'v + v'u$

$3\ln x + \frac{1}{x}(3x+1)$

$3\ln x + 3 + \frac{1}{x}$

b.) $f(x) = (x^4 + 3x^3 + 6)(2x - 1)$

$(4x^3 + 9x^2)(2x-1) + 2(x^4 + 3x^3 + 6)$

$8x^4 - 4x^3 + 18x^3 - 9x^2 + 2x^4 + 6x^3 + 12$

$10x^4 + 20x^3 - 9x^2 + 12$

c.) $f(x) = \frac{5x+3}{x^2+1}$

$\frac{5(x^2+1) - 2x(5x+3)}{(x^2+1)^2}$

$\frac{5x^2 + 5 - 10x^2 - 6x}{(x^2+1)^2} = \boxed{\frac{-5x^2 - 6x + 5}{(x^2+1)^2}}$

d.) $f(x) = \frac{x+2}{2e^x - 3}$

$\frac{1(2e^x - 3) - 2e^x(x+2)}{(2e^x-3)^2}$

$(2e^x - 3 - 2xe^x - 4e^x)/(2e^x-3)^2$

$\boxed{\frac{-2e^x - 3 - 2xe^x}{(2e^x - 3)^2}}$

like Ex 9c

Example 10 Find the derivative. If it is more convenient to rewrite the function first, do so.

a.) $f(x) = \sqrt{x}\,(4x^2 - 2x) = x^{\frac{1}{2}}(4x^2-2x)$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(4x^2-2x) + (8x-2)(x^{\frac{1}{2}})$

$2x^{3/2} - x^{1/2} + 8x^{3/2} - 2x^{\frac{1}{2}}$

$10x^{3/2} - 3x^{1/2} = \boxed{10\sqrt{x^3} - 3\sqrt{x}}$

b.) $f(x) = \frac{3x+4}{x^2-2} = \frac{3(x^2-2) - 2x(3x+4)}{(x^2-2)^2}$

$\frac{3x^2-6-6x^2-8x}{(x^2-2)^2} = \boxed{\frac{-3x^2-8x-6}{(x^2-2)^2}}$

c.) $f(x) = \frac{9}{\sqrt[3]{x^4}} = 9x^{-4/3}$

$f'(x) = -12x^{-7/3} = \boxed{\frac{-12}{\sqrt[3]{x^7}}}$

d.) $f(x) = \frac{3x^2+2x+1}{x^2} = 3 + 2x^{-1} + x^{-2}$

$f'(x) = -2x^{-2} - 2x^{-3}$

$\boxed{\frac{-2}{x^2} - \frac{2}{x^3}}$

Example 11 Find the following.

a.) Find d/dx[(ln x)(7x – 2)]

derivative

$\frac{1}{x}(7x-2) + 7(\ln x)$

$7 - \frac{2}{x} + 7\ln x$

b.) If $s(t) = (4t^2 - 1)^2$, find ds/dt.

$(4t^2-1)(4t^2-1)$

$8t(4t^2-1) + 8t(4t^2-1)$

$2[8t(4t^2-1)]$

$2[32t^3 - 8t]$

$64t^3 - 16t$

c.) If $A = \pi r^2$, find $\left.\frac{dA}{dr}\right|_{r=3}$

find derivative at $r=3$

$\frac{dA}{dr} = 2\pi r$

$r=3$

$2\pi(3)$

6π