Topic 1 Day 2 – Geometric Sequences and Series

Geometric Sequence a sequence in which each element, after the first, is obtained by <u>multiplying</u> the previous element by a fixed number (called the common ratio)

<u>FORMULA to find the nth term</u>

Geometic sequence:	$u_n = u_1 \bullet r^{n-1}$	where $r = \frac{u_2}{u_1}$ or $\frac{u_n}{u_{n-1}}$	*IB Formula Sheet*
u_n = nth term	u_1 = first term	n = # of terms	r = common ratio

ex. Find the tenth term in the sequence 2, 6, 18, 54, 162, ...

$r = 3$ (multiply each time)

$u_1 = 2$ $\quad u_n = u_{10}$

$u_{10} = u_1 \cdot r^{n-1}$

$2 \cdot (3)^{10-1}$

$2 \cdot 3^9$

$2 \cdot 19{,}683 = \boxed{39{,}366}$

ex. Find the eleventh term in the sequence $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots\ldots$

$r = -\frac{1}{2}$ $\quad u_1 = 1$ $\quad u_n = u_{11}$

$u_{11} = u_1 \cdot r^{n-1}$

$= 1 \cdot \left(-\frac{1}{2}\right)^{11-1}$

$= 1 \cdot \left(-\frac{1}{2}\right)^{10}$

either ...

$1 \cdot \left(\frac{(-1)^{10}}{2^{10}}\right) = \frac{1}{1024}$

or

$\left(-\frac{1}{2}\right)^{10} = 9.76\ldots E^{-4}$

math → frac

$\frac{1}{1024}$

ex. A geometric sequence has a fifth term of 3 and a seventh term of 0.75. Find the first term, the common ratio, and the tenth term.

$u_5 = 3$ $\quad u_5 = u_1 r^{5-1} \Rightarrow u_5 = u_1 r^4$

$u_7 = 0.75$ $\quad u_7 = u_1 r^{7-1}$ $\quad u_7 = u_1 r^6$

divide to find r

$\frac{u_7}{u_5} = \frac{u_1 r^6}{u_1 r^4} \Rightarrow \frac{0.75}{3} = r^2$ $\quad .25 = r^2$ $\quad \boxed{r = .5 \text{ or } -.5}$

$u_5 = u_1 r^{5-1}$

$3 = u_1 (.5)^4$

$\boxed{u_1 = 48}$

($r = -.5$ gives same)

$u_{10} = 48(.5)^{10-1}$

$\boxed{u_{10} = .09375}$

if $r = -.5$

$\boxed{u_{10} = -.09375}$

ex. Find the number of terms in the geometric sequence 0.25, 0.75, 2.25, ..., 44286.75.

$r = \frac{0.75}{0.25} = 3$

$u_1 = 0.25$

$u_n = 44286.75$

$u_n = u_1 \cdot r^{n-1}$

$44286.75 = 0.25\,(3)^{n-1}$

$177147 = 3^{n-1}$

Graph + find intersection

$y_1 = 177147$

$y_2 = 3^{x-1}$

$\boxed{x = 12}$

Geometric Sequences and Series

ex. A car whose original value was \$34000 loses 15% of its value each year.

a. Write a geometric sequence that gives the year by year value of the car.

loses a % each year

geometric $r = (1 - .15) = .85$

$U_n = 34000(.85)^{n-1}$

b. Find the value of the car after 6 years.

$U_n = 34000(.85)^{6-1}$

$34000(.85)^5 =$ \$15085.98

c. After how many years will the value of the car fall below \$10000 ?

$10000 = 34000(.85)^{n-1}$

$\frac{10000}{34000} = .85^{n-1}$

$5/17 = .85^{n-1}$

Use calc to solve

$y_1 = 5/17$

$y_2 = .85^{n-1}$

$n = 8.5$

so 9 yrs

ex. The number of people in a small town increases by 2% each year. If the population at the start of → 1970 was 12500, what is the predicted population at the start of 2010 ?

increase a % each year

geometric $r = (1 + .02) = 1.02$

$U_1 = 12500$

1970 = year 1

1969 = year 0

2010 − 1969 = year 41 $n = 41$

$U_n = (12500)(1.02)^{41-1}$

$= 12500(1.02)^{40}$

$= 27600$ 3 sf

GEOMETRIC SERIES: If the terms of a geometric sequence are added together, the result is known as series.

FORMULA to find the Sum

Geometic series: $S_n = \frac{u_1(1-r^n)}{1-r}$ when $|r| < 1$ or $S_n = \frac{u_1(r^n-1)}{r-1}$ when $|r| > 1$

IB Formula Sheet

u_n = n^{th} term u_1 = first term n = # of terms r = common ratio

the $|r|$ part is not in Formula Booklet

ex. Find the sum of the first 9 terms of the series 2 + 4 + 8 + 16 + …………

$r = 2$ so $|2| > 1$

$n = 9$

$U_1 = 2$

$S_9 = \frac{2(2^9 - 1)}{2 - 1} = \frac{2(511)}{1} = \boxed{1022}$

note: you get the same answer using the other formula!

Geometric Sequences and Series

ex. Find the sum of the first 12 terms of the series $24 + 18 + \frac{27}{2} + \frac{81}{8} + \ldots\ldots\ldots$

$r = \frac{18}{24} = .75$ $\quad |.75| < 1$ $\quad S_{12} = \frac{24(1-.75^{12})}{1-.75}$

$n = 12$

$u_1 = 24$

$\frac{24(\text{long decimal})}{.25} = 92.959\ldots$

93.0 3 s.f.

(can't change to fraction)

ex. The 2nd term of a geometric series is –30 and the sum of the first two terms is –15. Find the first term and the common ratio.

$u_2 = -30$

$u_1 + u_2 = -15$

$u_1 + (-30) = -15$

$u_1 = 15$

$r = \frac{u_2}{u_1} = \frac{-30}{15} = -2 = r$

ex. A company is offering Abid a job with an initial annual salary of \$28000 and a 4% raise each year after that. This 4% raise continues every year.

a. Find what Abid's salary will be after five years.

raise = increase 4%

$r = 1.04$

$u_1 = 28{,}000$

$n = 5$

$28000(1.04)^{5-1}$

\$ 32,756.04

b. Calculate the amount of money Abid will have earned after 15 years.

Sum

$u_1 = 28000$

$n = 15$

$r = 1.04$

$|1.04| > 1$

$\frac{28000(1.04^{15}-1)}{1.04-1}$

$\frac{28000(1.04^{15}-1)}{.04} = \$560{,}660.45$

Geometric Sequences and Series, Sigma Notation

SIGMA NOTATION

$\sum$ stands for "the sum of…."

This means that the expression $\sum_{i=1}^{n} u_i = u_1 + u_2 + u_3 + \ldots\ldots + u_{n-1} + u_n$

ex. Find $\sum_{i=1}^{5} 5(2)^{n-1}$

method 1: find all 5 terms and add

$i=1 \quad 5(2)^{1-1} = 5(2)^0 = 5$

$i=2 \quad 5(2)^{2-1} = 5(2)^1 = 10$

$i=3 \quad 5(2)^{3-1} = 5(2)^2 = 20$

$i=4 \quad 5(2)^{4-1} = 5(2)^3 = 40$

$i=5 \quad 5(2)^{5-1} = 5(2)^4 = 80$

Sum:
$5+10+20+40+80$
$\boxed{155}$

method 2: use one of the sum formulas

$$S_n = \frac{u_1(r^n - 1)}{r-1}$$

find the 1st and 2nd terms as done above

$\boxed{u_1 = 5}$

$u_2 = 10$

so $r = \frac{10}{5} = 2$ $\boxed{r=2}$

$n = 5$ terms
(1 to 5)

$$\frac{5(2^5-1)}{2-1} = \frac{5(31)}{1} = \boxed{155}$$

Good method when n is large!