7.2 Tangent Line and Derivative of x^n (Day 2)

From your investigation, you hopefully discovered our first rule for derivatives.

Power Rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$, where $n \in$ Real Numbers

Example 5: Use the power rule to find the derivative of each function below.

a.) $f(x) = x^{12}$

$f'(x) = 12x^{11}$

b.) $f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3x^{-4} = \frac{-3}{x^4}$

c.) $f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} =$

$\frac{1}{2}\frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

d.) $f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}x^{-4/3} =$

$-\frac{1}{3}\frac{1}{\sqrt[3]{x^4}} = \frac{-1}{3\sqrt[3]{x^4}}$

Definition: The process of finding the derivative of a function is called **differentiation.**

More derivative Rules:

Think about y = c, where c is a constant. Think about the slope.

Constant Rule: If f(x) = c, where c is any real number, then $f'(x) = 0$.

The Constant Multiple Rule: If c is a constant and f is a differentiable function, then
$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

The Sum Rule: If f and g are both differentiable, then
$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

The Difference Rule: If f and g are both differentiable, then
$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Example 6: Differentiate each function.

a.) $f(x) = 4x^3 + 2x^2 - 3$

$f'(x) = 12x^2 + 4x$

b.) $f(x) = 3\sqrt[5]{x} + 8 = 3x^{1/5} + 8$

$f'(x) = \frac{3}{5}x^{-4/5} = \frac{3}{5x^{4/5}}$ or $\frac{3}{5\sqrt[5]{x^4}}$

c.) $f(x) = (x-2)(x+4) = x^2 + 2x - 8$

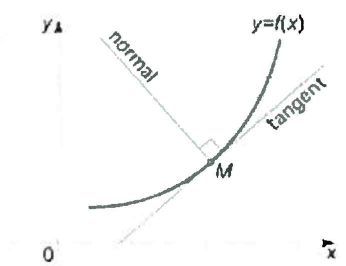
$f'(x) = 2x + 2$

d.) $f(x) = \frac{4x^3 + 2x^2 - 3}{x} = 4x^2 + 2x - 3x^{-1}$

$f'(x) = 8x + 2 + 3x^{-2}$

$= 8x + 2 + \frac{3}{x^2}$

Equations of Tangent and Normal Lines



Definition: A <u>tangent line</u> is a line that touches a curve at a single point on a curve. The slope of the tangent line at a given *x*-value can be found by finding the value of the derivative of the function at that given *x*-value.

Definition: A <u>normal line</u> is a line at a point on a curve that is perpendicular to the tangent line at that point. The slope of the normal line at a given *x*-value can be found by finding the opposite reciprocal value of the derivative of the function at that given *x*-value.

Example 7: Write an equation for each line.

a.) The tangent line to the curve $f(x) = x^2 + 1$ at $\boxed{(1,2)}$. So ... $x = 1$

$f'(x) = 2x$ $\quad y - y_1 = m \cdot (x - x_1)$

$m = f'(1) = 2(1) = 2$ $\quad y - 2 = 2(x - 1)$ ← IB normally leaves it in pt/slope form!

b.) The normal line to the curve $f(x) = 2\sqrt{x}$ when $x = 9$.

if $x = 9$ then $y = 2\sqrt{9} = 6$ $\quad$ pt $(9,6)$ $\quad f(x) = 2x^{1/2}$

$f'(x) = 1x^{-1/2} = \frac{1}{\sqrt{x}}$ $\quad y - y_1 = m(x - x_1)$

$y - 6 \;\; -3(x - 9)$

$m = f'(9) = \frac{1}{\sqrt{9}} = \frac{1}{3}$ $\quad$ perp slope $= -3$

c.) The tangent **and** normal lines to the curve $f(x) = x + \frac{27}{2x^2}$ when $x = 3$. $\quad$ so $f(x) = 3 + \frac{27}{2(9)} = \frac{9}{2}$

pt $(3, 9/2)$ $\quad f(x) = x + \frac{27}{2}x^{-2}$

$f'(x) = 1 - 27x^{-3} = 1 - \frac{27}{x^3}$

$m = f'(3) = 1 - \frac{27}{3^3} = 1 - \frac{27}{27} = 0$

tangent line

$y - y_1 = \;\; (x - x_1)$

$y - 9/2 = 0(x - 3)$ or $y = 9/2$ (Hor. Line)

normal line

perp slope = undefined ... so vertical line

$x = 3$

d.) The tangent to $f(x) = x^3 - 3x^2 - 13x + 15$ that is parallel to the tangent at $(4, -21)$.

* one tangent goes through $x=4$ with a certain slope. Find another x with the same slope!

$f'(x) = 3x^2 - 6x - 13$

$f'(4) = 3(4)^2 - 6(4) - 13 = 11$

Let $f'(x) = 11$

$11 = 3x^2 - 6x - 13$

$0 = 3x^2 - 6x - 24$

$0 = x^2 - 2x - 8$

$0 = (x - 4)(x + 2)$

$x = 4, x = -2$

$f(-2) = 21$

$m = 11$ $\quad$ pt$(-2, 21)$

$y - 21 = 11(x + 2)$

something like this