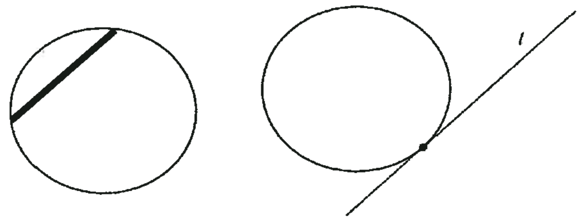
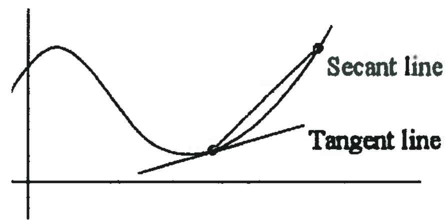
7.2 Tangent Line and Derivative of x^n (Day 1)

Definition: Recall from Geometry your knowledge about **secant lines** and **tangent lines.**

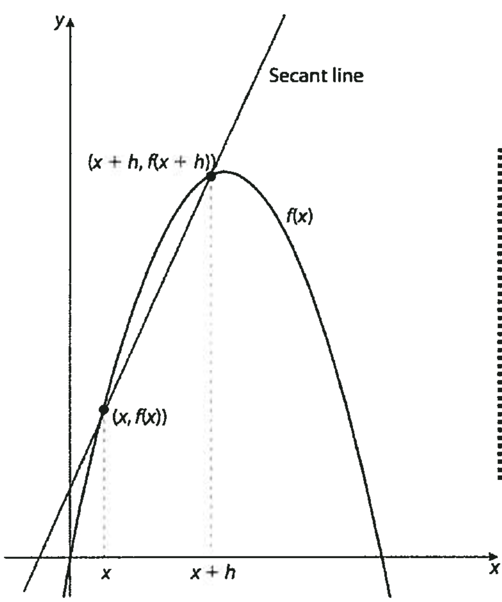
Now let's use it for Calculus





Gradient (slope) of a Secant Line

Definition: The average rate of change of a function is found by finding the slope of a secant line.



Definition: The gradient of a secant line is the same as the slope between two points on the line.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

(Also known as the difference quotient from Pre-Calculus)

slope

Example 3: Write an expression for the gradient of a secant line for f(x) = x² + 1. Simplify your answer.

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2+1] - [x^2+1]}{h}$$

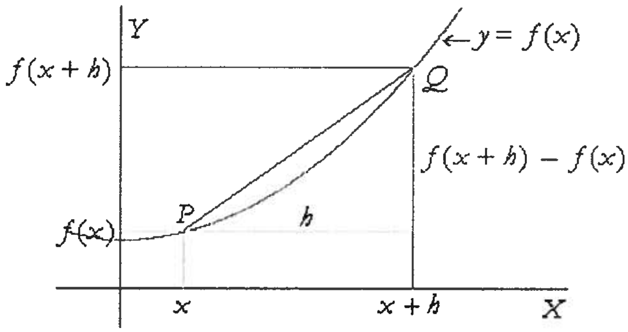
Simplify! Everything with an "h" must cancel!

$$\frac{[x^2+2xh+h^2+1] - [x^2+1]}{h} = \frac{x^2+2xh+h^2+1-x^2-1}{h}$$

$$\frac{2xh+h^2}{h} = \boxed{2x+h}$$

think e

Gradient (Slope) of a tangent line and the derivative



If the distance between the x-values of two points P and Q (represented by h) becomes closer and closer to zero then it becomes a tangent line.

That slope is represented by

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition: **This function is known as the <u>DERIVATIVE.</u>**

*Note that the derivative can be symbolized by $f'(x)$ or y' or $\frac{dy}{dx}$.

slope

@ (b)

Example: **Use the definition of derivative to find the derivative of $f(x) = 3x^2 + x$ and hence find the gradient of the tangent line when $x = 4$ (find the value of $f'(4)$)**

derivative

Definition of Derivative:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

*Same start as last example...

$$\frac{dy}{dx} = f'(x) = y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{[3(x+h)^2 + (x+h)] - [3x^2+x]}{h} =$$

$$\lim_{h \to 0} \frac{[3(x^2+2xh+h^2) + x+h] - [3x^2+x]}{h} = \frac{3x^2+6xh+3h^2+x+h-3x^2-x}{h}$$

$$\lim_{h \to 0} \frac{6xh+3h^2+h}{h} = \lim_{h \to 0} 6x+3h+1 \Rightarrow 6x+3(0)+1$$

$$= 6x+1 \quad @$$

<u>Now</u> $f'(4) = 6(4)+1 = 25$ (b)

Hmwk D

→Do the Investigation on page 203 in textbook.