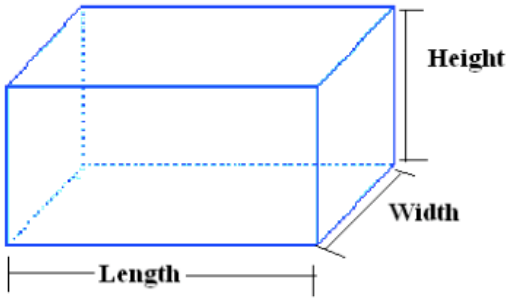
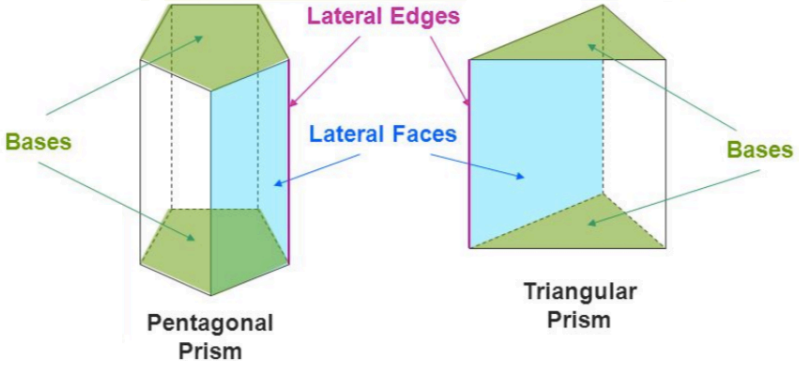
## Section 6.6 The Geometry of Three-Dimensional Solids

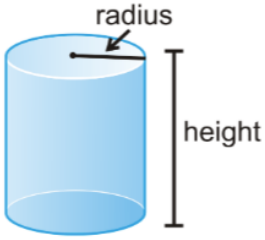
**The following formulas are given to you in your formula booklet...**

| | |
|---|---|
| Area of the curved surface of a cylinder | $A = 2\pi rh$, where $r$ is the radius, $h$ is the height |
| Surface area of a sphere | $A = 4\pi r^2$, where $r$ is the radius |
| Area of the curved surface of a cone | $A = \pi rl$, where $r$ is the radius, $l$ is the slant height |
| Volume of a pyramid | $V = \frac{1}{3}Ah$, where $A$ is the area of the base, $h$ is the vertical height |
| Volume of a cuboid | $V = l \times w \times h$, where $l$ is the length, $w$ is the width, $h$ is the height |
| Volume of a cylinder | $V = \pi r^2 h$, where $r$ is the radius, $h$ is the height |
| Volume of a sphere | $V = \frac{4}{3}\pi r^3$, where $r$ is the radius |
| Volume of a cone | $V = \frac{1}{3}\pi r^2 h$, where $r$ is the radius, $h$ is the vertical height |
| Volume of a prism | $V = Ah$, where $A$ is the area of cross-section, $h$ is the height |

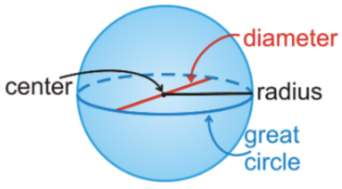



**Cuboid**

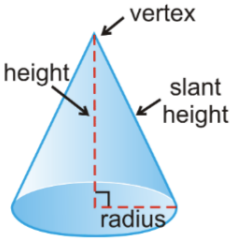


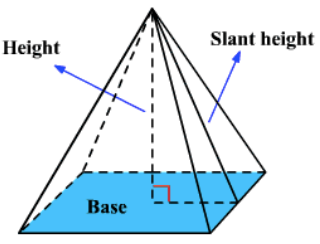

**Cylinder**




**Sphere**

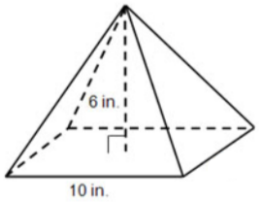



**Cone**




**Pyramid**

Example: Find the surface area and volume of the given pyramid correct to 3 sig figs. Let the units be in mm.




Surface Area:
In this case... the area of the square base and the 4 triangular sides.

First find the slant height.
This is the height of each triangle:

$$6^2 + 5^2 = l^2 \rightarrow l = \sqrt{61}$$

Area of 4 triangles: $4\left({}^1\!/_2\,(10)\left(\sqrt{61}\right)\right)$

156 mm$^2$

Area of base: (10)(10) = 100 mm$^2$

Surface Area = 256 mm$^2$

Volume:
The height of the pyramid is 6 mm.

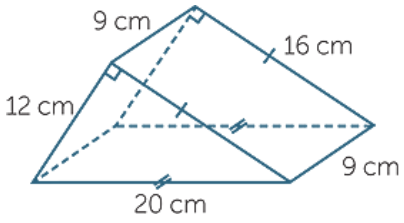
The area of the base is
(10)(10) = 100

So the volume is...

$$V = {}^1\!/_3\,Ah = {}^1\!/_3\,(100)(6)$$

$$= {}^1\!/_3\,(600) = 200 \text{ mm}^3$$

Example: Find the surface area and volume of the given prism correct to 3 sig figs.




Surface Area:
In this case we have two congruent triangular bases and three rectangular faces.

Area of 2 bases:

$$2\left({}^1\!/_2\,bh\right) = 2\left({}^1\!/_2\,(12)(16)\right)$$

= 192 cm$^2$

Area of faces:

$$(12)(9) + (9)(16) + (20)(9)$$

= 108 + 144 + 180 = 432 cm$^2$

Total Surface Area:
192 + 432 = 624 cm$^2$

Volume:
In this case the cross-section is a base.
So the area of the base is:

$${}^1\!/_2\,bh = {}^1\!/_2\,(12)(16) = 96$$

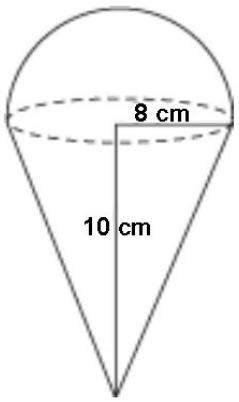
The height of the prism is the distance between the two bases... so $h$ = 9 cm

So the volume is...

$$V = Ah = (96)(9)$$

864 cm$^3$

Find the surface area and volume of the given shape correct to 3 sig figs.




Surface Area:
Cone (minus top) + Half a Sphere

Cone (Slanted Area Only):
Slant Height:

$$8^2 + 10^2 = l^2 \rightarrow l = \sqrt{164}$$

$$A = \pi rl = \pi(8)\left(\sqrt{164}\right) = 322$$

Half a Sphere:

$$A = {}^1\!/_2\,(4\pi(8)^2) = 128\pi = 402$$

Total Surface Area:
322 + 402 = 724 cm$^2$

Volume:
Cone + Half a Sphere

Cone:

$$V = {}^1\!/_3\,\pi r^2 h = {}^1\!/_3\,\pi(8)^2(10)$$

= 670 cm$^3$ (3 sig figs)

Half a Sphere:

$$V = {}^1\!/_2\left({}^4\!/_3\,\pi(8)^3\right) = 1072$$

Total Volume: 1742 cm$^3$
or 1740 cm$^3$