Section 6.4 The Cosine Rule

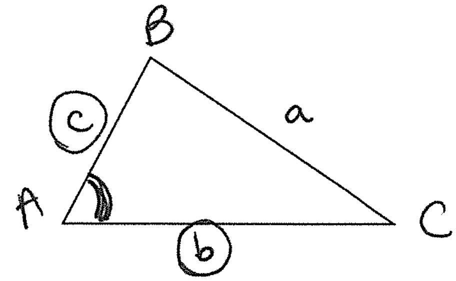
Sometimes you cannot use the Law of Sine. If you don't have a side and an angle opposite it then you may have to use the Law of Cosine.

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

or

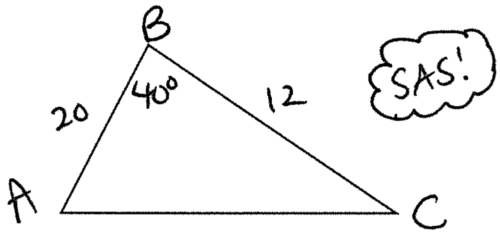
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



Note: When computing keep as much as you can in your calculator to ensure the most accurate answer. Also make sure you are in DEGREE mode!

Example: Solve the following triangles. Round answers to ~~the nearest tenth~~. 3 sig figs

1) B=40°, $a = 12$, $c = 20$



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 12^2 + 20^2 - 2(12)(20)\cos 40°$$

$$b^2 \approx 176.3$$

$$b = 13.3$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(13.3^2 + 20^2 - 12^2)}{(2 \cdot 13.3 \cdot 20)}$$

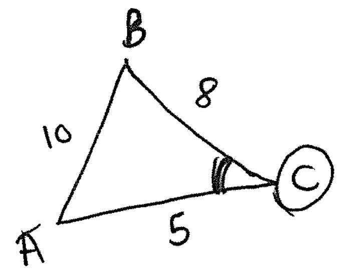
$$\cos A \approx 0.8137\ldots$$

$$\cos^{-1}(ANS) = A \qquad A \approx 35.5°$$

$$C = 180° - 40° - 35.5°$$
$$C = 104.5°$$

2) $a = 8$, $b = 5$, $c = 10$

Start with the largest angle first: C



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(8^2 + 5^2 - 10^2)}{(2 \cdot 8 \cdot 5)}$$

$$\cos C = -0.1375$$

$$\cos^{-1}(ANS) = C \qquad C \approx 97.9°$$

Now I can switch to Law of Sine...

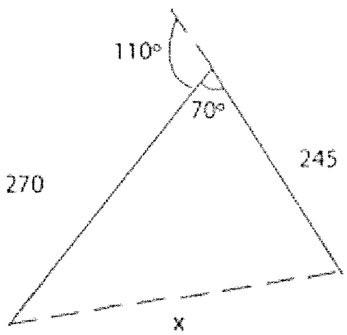
$$\frac{8}{\sin A} = \frac{10}{\sin 97.9°}$$

$$\sin A = \frac{8 \cdot \sin 97.9°}{10} \approx 0.79\ldots$$

$$\sin^{-1}(ANS) = A \qquad A \approx 52.4°$$

$$B = 180° - 97.9° - 52.4° = 29.7°$$

Example 3: To approximate the length of a lake, a surveyor starts at one end of the lake and walks 245 meters. He then turns 110° and walks 270 meters until he arrives at the other end of the lake. Approximately how long is the lake? If the surveyor could walk on water at a rate of 0.5ms^{-1} (meters per second), how long would it take to the nearest second to walk across the lake?



Remember… a straight line is 180°

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 245^2 + 270^2 - 2(245)(270)\cos 70$$

$$a^2 \approx 87{,}675.74$$

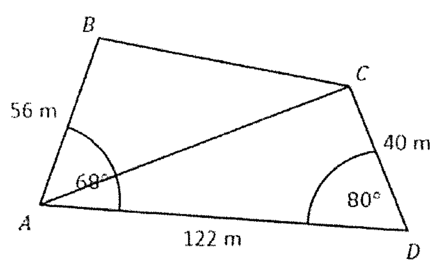
$$a = 296.1$$

So the lake is 296 meters (using 3 sf)

As far as how long... Remember distance = (rate)(time). So if I divide by rate I will have time...

(distance) / (rate) = time → 296 / 0.5 = 592 seconds

The quadrilateral ABCD is shown below, where AB = 56 m, AD = 122 m, CD = 40 m, and the angles BAD and ADC are 68° and 80° respectively. Find the following:



a) Find the length of AC
b) Find the angle DAC in degrees to the nearest tenth
c) Find the shortest distance between point B and AC
d) Find the length of the perimeter of triangle ABC

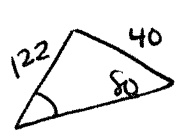
a) Use Cosine Rule since you have SAS:

$$(AC)^2 = 122^2 + 40^2 - 2(122)(40)\cos 80°$$

$$AC \approx 121.6 = 122 \text{ (to 3 sf)}$$

b) Use Cosine or Sine Rule

$$\frac{40}{\sin(DAC)} = \frac{122}{\sin 80°}$$



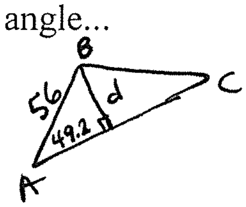
$$\sin(DAC) = \frac{40 \cdot \sin 80°}{122} \approx 0.32\ldots$$

$$\sin^{-1}(ANS) \rightarrow DAC \approx 18.8°$$

c) The shortest distance d would be at a right angle...

$$x = 68° - 18.8° = 49.2°$$

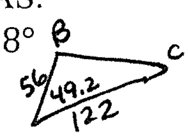
$$\sin 49.2° = \frac{d}{56} \rightarrow d \approx 42.4$$



d) Find BC first. Use Cosine Rule since I have SAS.

$$(BC)^2 = 56^2 + 122^2 - 2(40)(122)\cos 18.8°$$

$$BC \approx 93.7$$



Perimeter: 56 + 93.7 + 122 = 271.7 → 272 meters