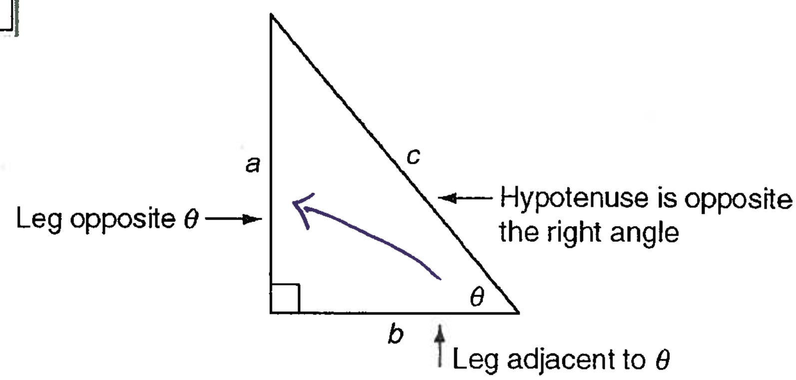
Section 6.1 *Advanced Right-Angled Trigonometry*

A **trigonometric ratio** compares the lengths of two sides of a right triangle. The values of the ratios depend upon one of the acute angles of the triangle, denoted by θ.

> Sine is **O**pposite over **H**ypotenuse,
> **C**osine is **A**djacent over **H**ypotenuse,
> **T**angent is **O**pposite over **A**djacent.

Use **SOHCAHTOA** to remember the relationships between the sides of a right triangle that correspond to the trigonometric ratios sine, cosine, and tangent.



$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

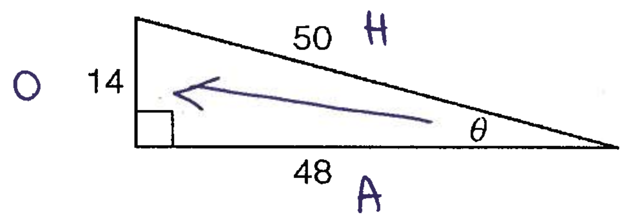
$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{b}$$

Example: Find the values of the trigonometric functions for θ; reduce if needed.

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{14}{50} = \frac{7}{25}$$

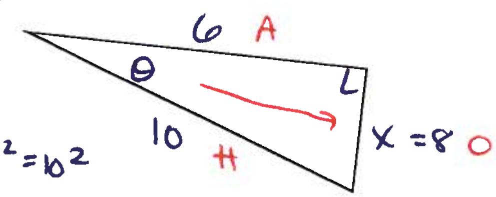
$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{48}{50} = \frac{24}{25}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{14}{48} = \frac{7}{24}$$



Example 1: Find the value of the sine, cosine, and tangent functions for θ.

1.



$6^2 + x^2 = 10^2$

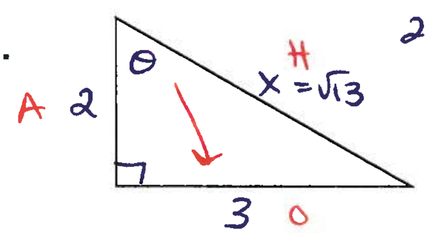
$x^2 = 64 \quad x = 8$

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{8}{6} = \frac{4}{3}$

2.



$2^2 + 3^2 = x^2$

$13 = x^2$

$x = \sqrt{13}$

$\sin\theta = \frac{3}{\sqrt{13}}$ or $\frac{3\cdot\sqrt{13}}{\sqrt{13}\cdot\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\cos\theta = \frac{2}{\sqrt{13}}$ or $\frac{2\cdot\sqrt{13}}{\sqrt{13}\cdot\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\tan\theta = \frac{3}{2}$

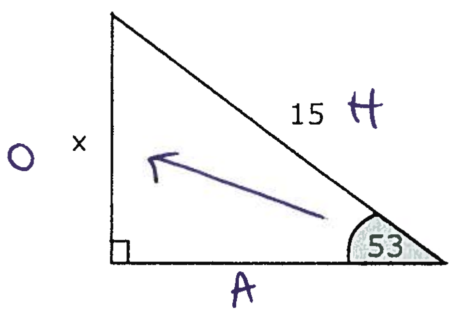
Always make sure you reduce your fractions, rationalize your denominators, and simplify any radicals.

Example: $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$ **Also, FYI...** $(4\sqrt{3})^2 = 4^2 \cdot (\sqrt{3})^2 = 16 \cdot 3 = 48$

In order to use trigonometry to find a missing side or angle, evaluate which sides are involved. That will determine which function to use. **Also make sure your calculator is in DEGREE mode; You will NEVER need to be in RADIAN mode.** The default mode on your graphing calc is RADIAN. Hit the MODE button to change if needed.

Example 2: Find the value of *x*. Be sure to give your answers to 3 sig figs.

a)



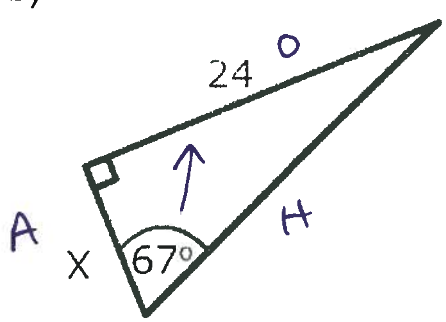
$$\sin 53° = \frac{x}{15}$$

$$15 \cdot \sin 53° = x$$

$$x \approx 11.9795 \ldots$$

$$x = 12.0$$

b)



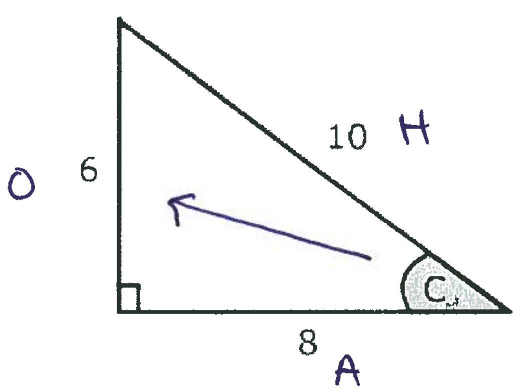
$$\tan 67° = \frac{24}{x}$$

$$x \cdot \tan 67° = 24$$

$$x = \frac{24}{\tan 67°}$$

$$x \approx 10.2$$

c)



In this case since we have all three sides I could use any of the functions. Let's use cosine since we haven't already...

In this case since we are looking for an angle so we need to use an inverse trigonometric function.

$$\cos C = \frac{8}{10}$$

$$\cos^{-1}\left(\frac{8}{10}\right) = C \quad \rightarrow \quad C \approx 36.9°$$

Extra 6.1 Examples:
Solve the following to 3 sig figs:

a) $\frac{\sin 33°}{4} = \frac{\sin 46°}{x}$

cross multiply!

$x \cdot \sin 33° = 4 \cdot \sin 46°$

$x = \frac{4 \cdot \sin(46°)}{\sin(33°)}$

$x \approx 5.28$

b) $6 \cos 48° = {}^{1}/_{3}\,(3x - 4)$

Either distribute the fraction or multiply to get rid of it...

$18 \cos 48° = 3x - 4$

$18 \cos(48°) + 4 = 3x$

$\frac{(18 \cos(48°) + 4)}{3} = x \qquad x \approx 5.35$

Geometry and Miscellaneous Review Topics

Properties of isosceles triangles:
2 congruent sides & 2 congruent angles (opposite of the congruent sides)

A linear pair (adjacent angles that together form a line) adds to 180°.
The height of any shape is a perpendicular segment.

Properties of Regular Polygons:
All sides and angles are congruent. A regular triangle is called an equilateral triangle. A regular quadrilateral is called a square.

Perimeter is the distance around the outside of a figure. In a circle this is called the circumference. Area is the measure of how much space is on a flat surface. In a 3D figure this is called surface area. The units are units^2. Volume is the measure of how much 3D space a figure takes up. Think of how much could fit inside it. The units of volume are units^3.

Keep in mind the formula distance = (rate)·(time). So rate = distance/time **and** time = distance/rate. This is NOT in your formula booklet. A rate can be given in any unit so just be sure you are finding it in the units asked for; if they ask for ft per second but you have minutes then you will need to convert. Sometimes the rates are written using negative exponents. For example, meters per second could be written as m/s (meters divided by seconds) or ms^{-1}.

If you're given a number in scientific notation, you can enter that in the calculator without converting it. For example... 4.53×10^{-4} can be entered in the calculator as 4.53*10^-4. If your calculator gives you a number in scientific notation it uses an E instead of a 10. So 4.53×10^{-4} would be given as 4.53E-4.

Remember, a negative exponent in scientific notation means to move the decimal place that many places to the left (it is a SMALL decimal). A positive exponent means to move the decimal place that many place to the right (it is a BIG number). Numbers in scientific notation are written in the form $a \times 10^k$ where $1 \le |a| < 10$ and k is a positive or negative integer (not a decimal). *a* can be positive or negative and can include a decimal but the absolute value of *a* must be smaller than 10.