Section 5.4 Solving Equations involving Combinations of Unfamiliar Functions

Many equations can be solved using algebraic, logarithmic, or trigonometric techniques. Some can be solved by doing a simple substitution method. But some can only be solved using graphing.

Substitution Example:

$2x^{\frac{2}{3}} + 7x^{\frac{1}{3}} = -3$

Let $y = x^{\frac{1}{3}}$

$2x^{\frac{2}{3}} + 7x^{\frac{1}{3}} + 3 = 0$

$2\left(x^{\frac{1}{3}}\right)^2 + 7x^{\frac{1}{3}} + 3 = 0$

$2y^2 + 7y + 3 = 0$

$(2y + 1)(y + 3) = 0$

$y = -1/2 \quad y = -3$

$x^{\frac{1}{3}} = -1/2 \quad x^{\frac{1}{3}} = -3$

$x = (-1/2)^3 = -1/8 \qquad x = (-3)^3 = -27$

SOLVING EQUATIONS USING YOUR GRAPHING CALCULATOR:

It is possible to solve equations with one variable using your Graphics Display Calculator and functions. There are basic steps that you can use to solve any equation using functions:

STEP 1: Put the left side of the equation into Y1 = in your calculator and the right side into Y2 = .

STEP 2: Graph the functions using your calculator. Be familiar with the type of function being graphed using each side of the equation. This will allow you to adjust your WINDOW to see more or less of the screen. If solving trigonometric equations, remember to be in DEGREE MODE and set your window using the degree values for the domain within which you are searching for solutions.

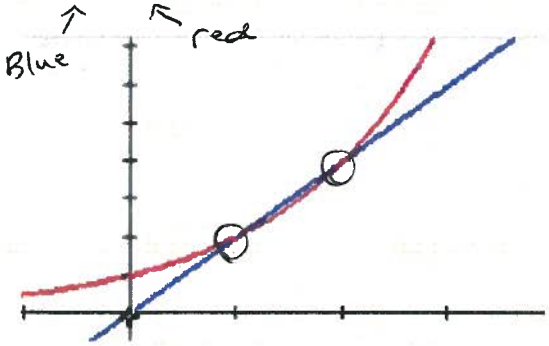
STEP 3: Find the point or points of intersection of the two functions. The x-coordinates will be the solutions to the equation.

STEP 4: Check your solutions to make sure that they make the original equation true.

EXAMPLES: Solve the following equations over the set of real numbers:

* Blue graph is always the left side of the equation

a) $2x = 2^x$



For this graph, I must use ZOOM BOX or ZOOM IN to see where they actually cross...

Solutions:

$x = 1, x = 2$

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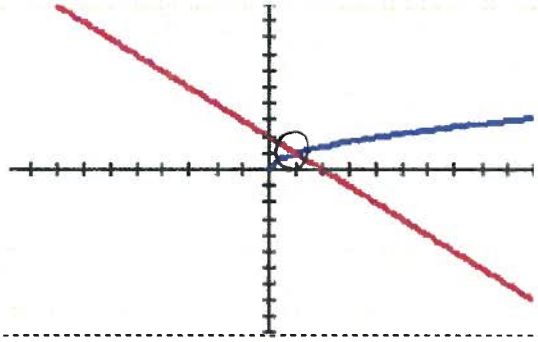
b) $\sqrt{x} = 2 - x$



Solution:

x = 1

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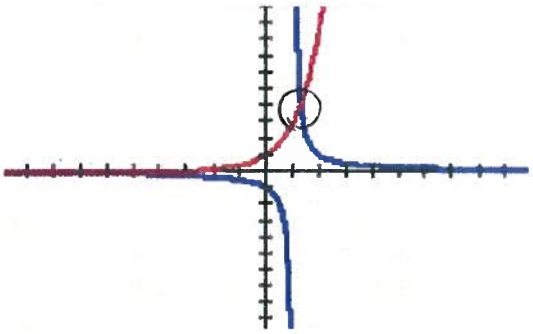
c) $\frac{1}{x-1} = 3^x$



In this graph it's hard to tell what is happening around the *x*-axis. Think about the graphs... The blue graph has a horizontal asymptote at *y* = 0 but the left side of its graph is BELOW the axis.

The red graph has a horizontal asymptote at *y* = 0 but the left side of its graph is ABOVE the axis. So there is NO intersection on the left.

Solution: *x* = 1.25

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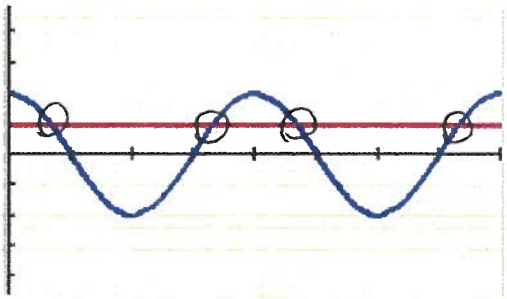
d) $2\cos 2x = 1$
Domain: $0° \leq x \leq 360°$



Make sure you are in DEGREE mode and use a *x*-min of 0 and a *x*-max of 360. I used a scale of 45. I went from –5 to 5 on the *y*-axis.

Solutions:

$x = 30°, 150°, 210°$, and $330°$