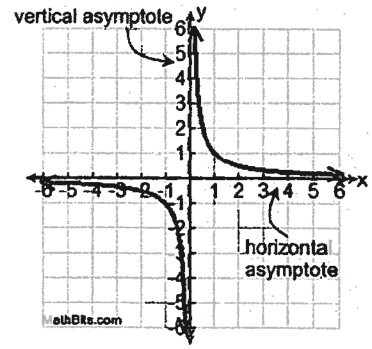
Section 5.2 Rectangular Hyperbolic Functions

A hyperbolic function consists of two curves called branches.
The parent function is $f(x) = \frac{1}{x}$.
The graph has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.
The domain in set notation is $D = \{x | x \in \mathbb{R}, x \neq 0\}$
The range in set notation is $R = \{y | y \in \mathbb{R}, y \neq 0\}$

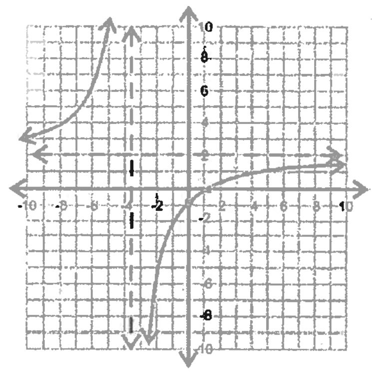


In this course we will concentrate on equations of the form $f(x) = \frac{a}{x-b} + c$

In these cases, a is the vertical stretch, b is the horizontal shift, and c is the vertical shift. The shifts will move the asymptotes as well.

The graph at the right has

- a horizontal asymptote at $y = 2$ (vertical shift up 2)
- a vertical asymptote at $x = -4$ (horizontal shift left 4)



Assuming no vertical stretch, the new equation would be

$$f(x) = \frac{1}{x+4} + 2$$

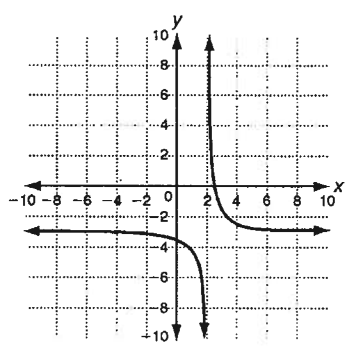
The domain in set notation is $D = \{x | x \in \mathbb{R}, x \neq -4\}$
The range in set notation is $R = \{y | y \in \mathbb{R}, y \neq 2\}$

Note: you can always shift the asymptotes of the parent function but you can also find the vertical asymptotes by setting the denominator equal to zero. The x-value you get is your vertical asymptote. You can find the horizontal asymptotes by plugging in large values of x. The y-value you get is your horizontal asymptote.

Example:

Graph $g(x) = \frac{1}{x-2} - 3$.

Vertical asymptote at $x = 2$.
Horizontal asymptote at $y = -3$.
Domain $= \{x | x \in \mathbb{R}, x \neq 2\}$
Range $= \{y | y \in \mathbb{R}, y \neq -3\}$



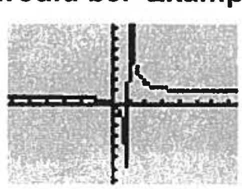
The graph of $f(x) = 1/x$ is translated 2 units right and 3 units down from the origin.

Note: When putting these in your graphing calculator, you MUST put your denominator in a parenthesis if you have a horizontal shift.

For example, the last example would look like

```
Plot1  Plot2  Plot3
\Y1=1/(X-2)-3
```

Note: Some graphing calculators connect the two branches in the spot where the vertical asymptote would be. Example:



This graph has a vertical asymptote at $x = 1$

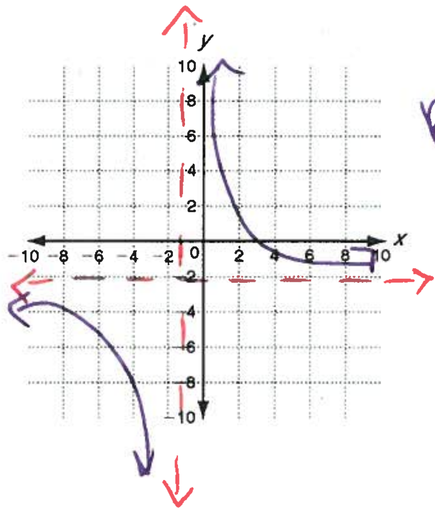
Example: Describe the transformation of $f(x) = \frac{1}{x}$. Then sketch each graph and identify the asymptotes.

1. $g(x) = \frac{1}{x+1} - 2$

Vertical asymptote: $x = -1$

Horizontal asymptote: $y = -2$

Transformation: left 1, down 2



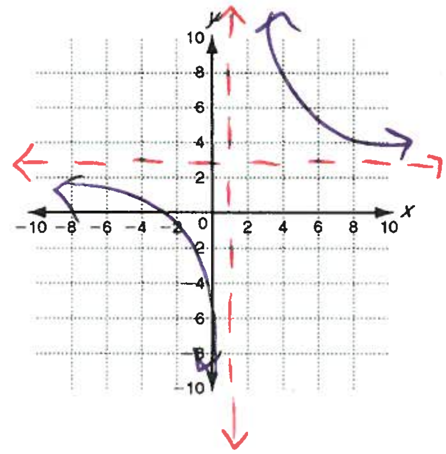
Rough Sketch

2. $g(x) = \frac{1}{x-1} + 3$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 3$

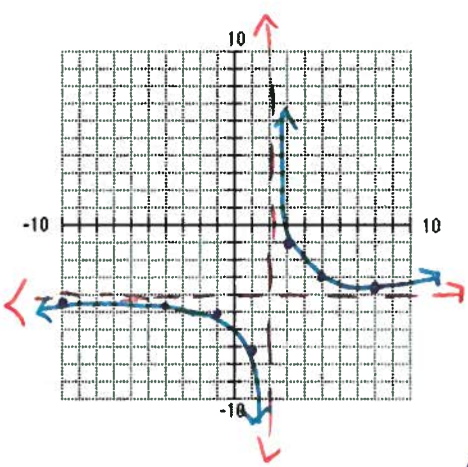
Transformation: right 1, up 3



Example: Draw the graph of each of the following below. Be sure to clearly show the points you plot.

(a) $y = \frac{3}{x-2} - 4$

This graph will have a vertical stretch of 3. It will have a vertical asymptote at $x = 2$ (right) and a horizontal asymptote at $y = -4$ (down)



x	y
-10	-4.25
-4	-4.5
-1	-5
1	-7
3	-1
5	-3
8	-3.5

Find fairly nice numbers to plot

(b) $y = \frac{4x}{x+2}$

Think of this function as

$$y = \frac{4x+0}{x+2} = \frac{4x+8-8}{x+2} = \frac{4(x+2)-8}{x+2} =$$

$$= \frac{4(x+2)}{(x+2)} - \frac{8}{(x+2)} = 4 + \frac{-8}{x+2}$$

So this graph will have a vertical flip and stretch of 8. It will be moved left 2 and up 4.

$y = 4$ (hor asym)
$x = -2$ (ver asym)

x	y
-10	5
-6	6
-4	8
-1	-4
0	0
2	2
6	3