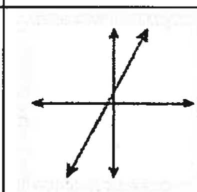
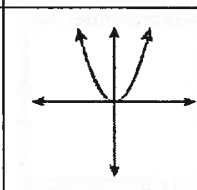
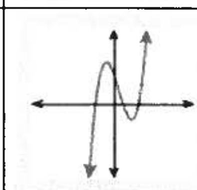
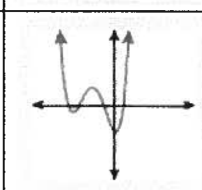
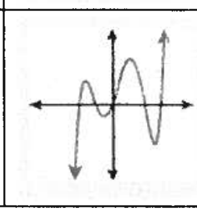
Section 5.1 Higher Order Polynomial Functions

The graphs of polynomial functions are classified by the degree (highest exponent) of the polynomial. Each graph, based on the degree, has a distinctive shape and characteristics.

A **turning point** is where a graph changes from increasing to decreasing or from decreasing to increasing. This point corresponds to a *local maximum* or *minimum.* Your book refers to maximums as humps (I prefer to say peaks) and to minimums as valleys. A **point of inflection** occurs between extreme points and can only be estimated in this course.

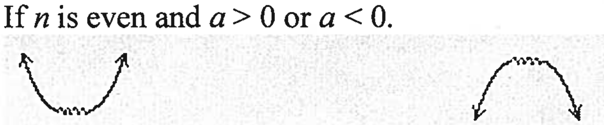
A polynomial function of degree n has at most n x-intercepts (roots, solutions, etc), at most $n-1$ Turning Points (peaks and valleys), and at most $n-2$ Points of Inflection.

Name	**Equation**	**Graph**
Linear: Degree 1	$y = ax + b$ No Turning Points At most 1 x-intercept	
Quadratic: Degree 2	$y = ax^2 + bx + c$ 1 Turning Point At most 2 x-intercepts	
Cubic: Degree 3	$y = ax^3 + bx^2 + cx + d$ At most 2 Turning Points At most 3 x-intercepts	
Quartic: Degree 4	$y = ax^4 + bx^3 + cx^2 + dx + e$ At most 3 Turning Points At most 4 x-intercepts	
Quintic: Degree 5	$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ At most 4 Turning Points At most 5 x-intercepts	

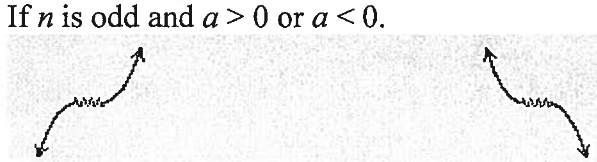
End behavior is a description of the values of the function as x approaches (gets closer to) infinity or negative infinity. The degree and leading coefficient of a polynomial function determine its end behavior. Both are helpful to know when you are graphing a polynomial function.

Note: The squiggly line represents whatever the middle of the graph does.

If n is even and $a > 0$ or $a < 0$.

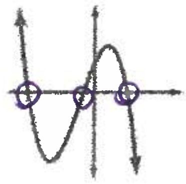


If n is odd and $a > 0$ or $a < 0$.



**Example: Identify the degree (even/odd) and end behavior for each function.
Then state the number of x-intercepts and inflection points.**

1.



Degree: odd

End Behavior: neg

x-intercepts: 3

Turning points: 2

2.

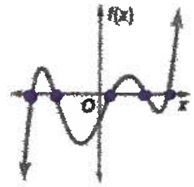
Degree: even

End Behavior: pos

x-intercepts: 4

Turning points: 3

3.



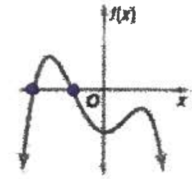
Degree: odd

End Behavior: pos

x-intercepts: 5

Turning points: 4

4.



Degree: even

End Behavior: neg

x-intercepts: 2

Turning points: 3

Example: Identify the following of each function.

5. $P(x) = 4x^3 + 8x^2 - 5$

Leading term & sign: $4x^3$

Sign and degree: pos / odd

End behavior: ↙ ↗ left down/right up

Max # of *x*-intercepts: 3

Max # of Turning Points: 2

6. $P(x) = -9x^6 + 2x^3 - x + 7$

Leading term & sign: $-9x^6$

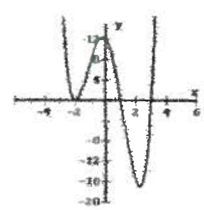
Sign and degree: neg / even

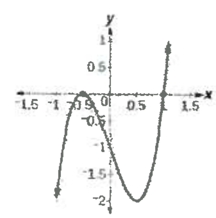
End behavior: ↙ ↘ both sides down

Max # of *x*-intercepts: 6

Max # of Turning Points: 5

Note: In graphs like the ones at the right, the zeros occurring at $x = -2$ and $x = -0.5$ are called a "double-zero" or a zero of multiplicity two. Notice how the graph just touches the *x* axis and stays on the same side of the *x*-axis. That occurs when you have the same zero (or factor) twice. You can have other occurances of multiple zeros (triple-zeros, etc) but that is beyond this course.

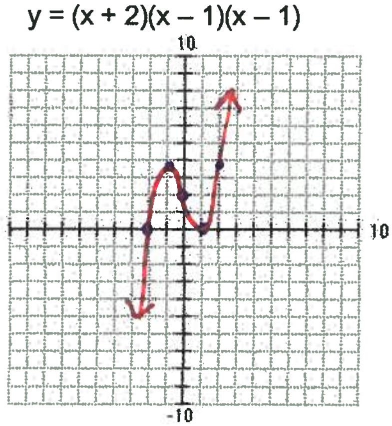




Examples: Graph the following cubic functions:

(a) $y = (x + 2)(x - 1)(x - 1)$

x	y
-3	-16
-2	0
-1	4
0	2
1	0
2	4
3	20



State the following:

$x+2=0$ $x-1=0$

$(0+2)(0-1)(0-1)$
$(2)(-1)(-1)$

X-Intercept(s) (Zeros): -2, 1   Y-Intercept(s): 2

Increasing Interval: $(-\infty, -1)$ $(1, \infty)$

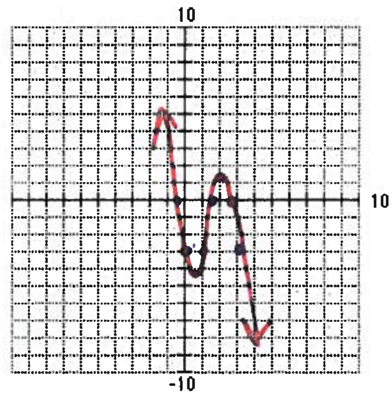
Decreasing Interval: $(-1, 1)$

Local Max(s): $(-1, 4)$ Local Min(s): $(1, 0)$

Approx Inflection Point(s): $(0, 2)$

(b) $y = -2x^3 + 8x^2 - 6x - 3$

x	y
-1	13
0	-3
1	-3
2	1
3	-3
4	-27



$-2(0)^3 + 8(0)^2 - 6(0) - 3$

State the following:

X-Intercept(s) (Zeros): -.336, 1.68, 2.66 Y-Intercept(s): -3

Increasing Interval: (0.451, 2.22)

Decreasing Interval: $(-\infty, 0.451)$ $(2.22, \infty)$

Local Max(s): (2.22, 1.23) Local Min(s): (0.451, -4.26)

Approx Inflection Point(s): (1.36, -1.38)

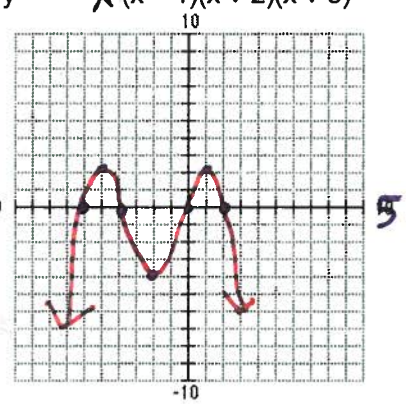
Examples: Graph the following quartic functions:

(c) $y = -x(x-1)(x+2)(x+3)$

$-x=0$ $x-1=0$ $x+2=0$ $x+3=0$

$-(0)(0-1)(0+2)(0+3)$

x	y
-4	-40
-3	0
-2	0
-1	-4
0	0
1	0
2	-40



State the following:

X-Intercept(s) (Zeros): 0, 1, -2, -3 Y-Intercept(s): 0

Increasing Interval: $(-\infty, -2.58)$ (-1, 0.581)

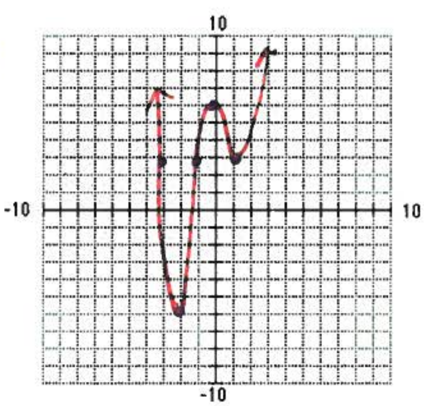
Decreasing Interval: (-2.58, -1) $(0.581, \infty)$

Local Max(s): (-2.58, 2.25) (0.581, 2.25) Local Min(s): (-1, -4)

Approx Inflection Point(s): (-1.82, -1.10) (0.152, -0.919)

(d) $y = x^4 + 2x^3 - 4x^2 - 2x + 6$

x	y
-3	3
-2	-6
-1	3
0	6
1	3



$0^4 + 2(0)^3 - 4(0)^2 - 2(0) + 6$

State the following:

X-Intercept(s) (Zeros): -2.90, -1.33 Y-Intercept(s): 6

Increasing Interval: (-2.28, -.219) $(1, \infty)$

Decreasing Interval: $(-\infty, -2.28)$ (-.219, 1)

Local Max(s): (-.219, 6.23) Local Min(s): (-2.28, -6.91) (1, 3)

Approx Inflection Point(s): (-1.36, -0.324) (0.455, 4.49)