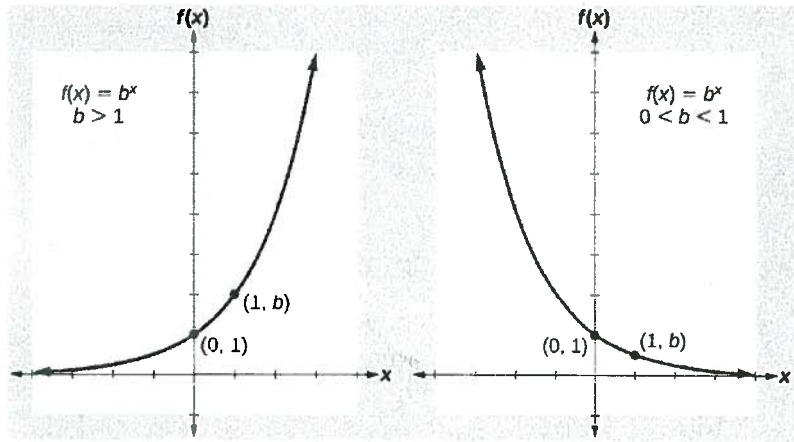
IB Math Studies
Section 4.8 & 4.9: Exponential Functions

$\mathbb{Z}$

An exponential function with the form
***f*(x) = bˣ, b > 0, b ≠ 1** ($f(x) = b^x,\ b > 0,\ b \neq 1$)
has these characteristics:

- horizontal asymptote: $y = 0$
- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- *x*-intercept: none
- *y*-intercept: $(0,1)$
- increasing if $b > 1$
- decreasing if $b < 1$



Since the domain is all Real numbers, it is possible to have negative and fractional exponents.

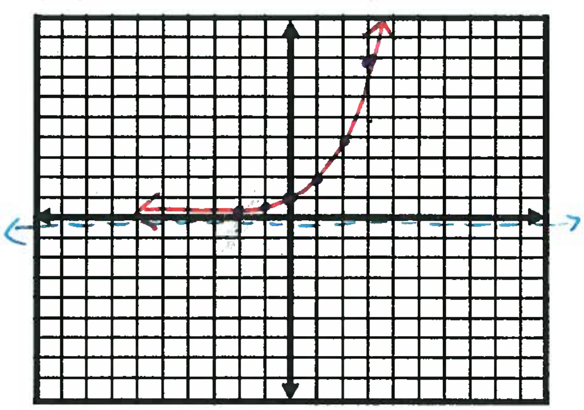
Example: $9^{-2} = \frac{1}{9^2} = \frac{1}{81}$ $\qquad 9^{\frac{1}{2}} = \sqrt{9} = 3$ $\qquad 9^{-\frac{1}{2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

$64^{\frac{1}{3}} = \sqrt[3]{64} = 4 \qquad 64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

Graphing Exponential Functions

ex. Graph $y = 2^x$

x	y
-2	.25
-1	.5
0	1
1	2
2	4
3	8



State the following:

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

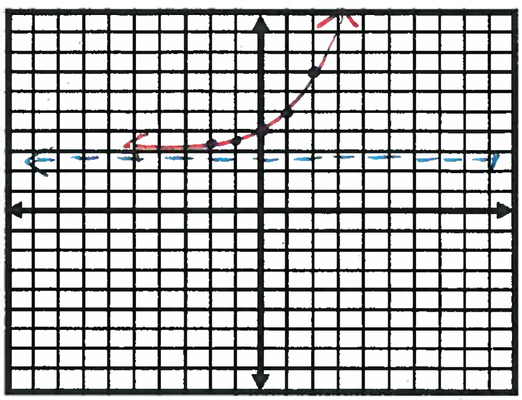
Intercepts: $(0, 1)$

INCREASING or DECREASING

Just like linear or quadratic functions, you can stretch, compress, flip, or shift exponential functions. When you shift a graph vertically, you shift the horizontal asymptote in the same way.

ex. Graph $y = 2^x + 3$ ← this is the last one... shifted up 1

x	y
−2	3.25
−1	3.5
0	4
1	5
2	7



State the following:

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

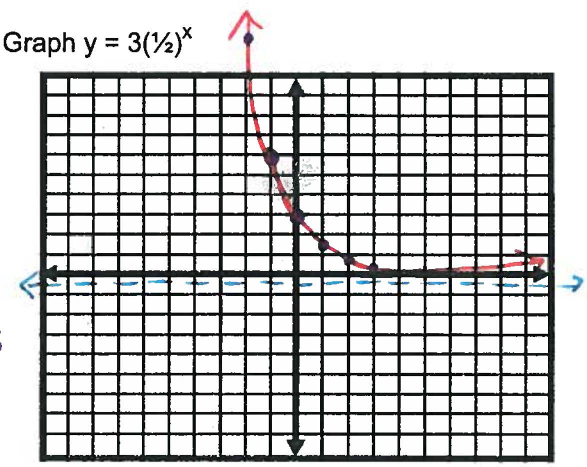
Asymptote: $y = 3$

Intercepts: $(0, 4)$

INCREASING or DECREASING

ex. Graph $y = 3(½)^x$

x	y
−2	12
−1	6
0	3
1	1.5
2	0.75
3	.375



State the following:

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

Intercepts: $(0, 3)$

INCREASING or DECREASING

SOLVING EXPONENTIAL EQUATIONS – 2 METHODS

1. **Solving Exponential Equations Graphically**
2. **Solving Exponential Equations Using logarithms**

Example: Solve the equation $2^x - 1 = 10$

1. **Solve Graphically:** The easiest way to solve this equation graphically is to use your TI-83 calculator and let $Y_1 = 2^x - 1$ and $Y_2 = 10$ and find the point of intersection of the two graphs, which is approx. ~~3.32~~. 3.46

2. **Solving Using Logarithms:** $2^x - 1 = 10$

Get the exponential expression by itself	$2^x = 11$
Take the log of both sides of the equation	$\log 2^x = \log(11)$
Using the properties of logs	$x \log 2 = \log(11)$
Divide both sides by the log 2	$\frac{x \log 2}{\log 2} = \frac{\log(11)}{\log(2)}$
	$x = 3.46$

Example: A radioactive element decays in such a way so that the amount present each year is 0.95 of the amount present the previous year. At the start of 1990, there were 50 mg of the element present.

a) Produce a table of values showing how much of the element is present at the start of each year from 1990 to 1998.

b) The rule for this situation is given by $N = k \times a^t$, where N mg is the amount of element present t years after the start of 1990. Find the value of a and k.

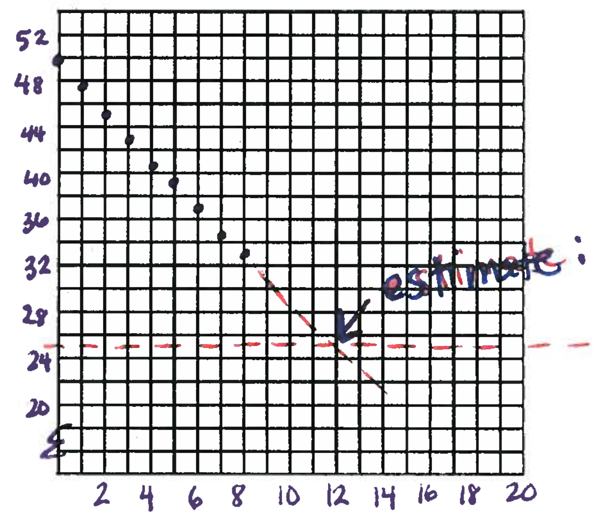
$N = 50 \cdot (.95)^t$

c) Sketch the graph of $N = k \times a^t$.

d) How long will it be before there is 25 mg of this element remaining?

Graph $y = 50(.95)^x$
$y = 25$
$\boxed{x = 13.5 \text{ yrs.}}$

yr	Amount
1990	50
1991	47.5
1992	45.125
93	42.86875
94	40.7253125
95	38.68904688
96	36.75459453
97	34.9168648
98	33.17162156



estimate: 12yrs.

Example: Five hundred dollars is deposited into a bank account that pays 5.40% interest per annum.

a) How much money will there be in the account at the end of 6 years if the interest is compounded annually?

$FV = PV\left(1 + \frac{5.4}{100}\right)^6 = 500(1.054)^6 = \$685.51$

b) How long will it be before the money in the account is doubled?

$1000 = 500(1.054)^x$ $x = 13.2$ yrs.

c) Show the behaviour of the compounding using a graph.