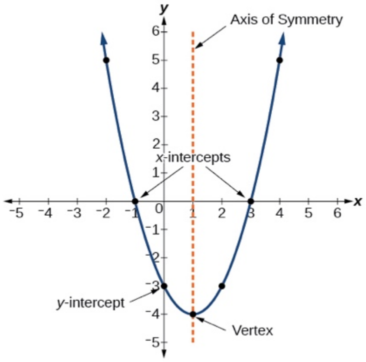
Section 4.7 Quadratic Models



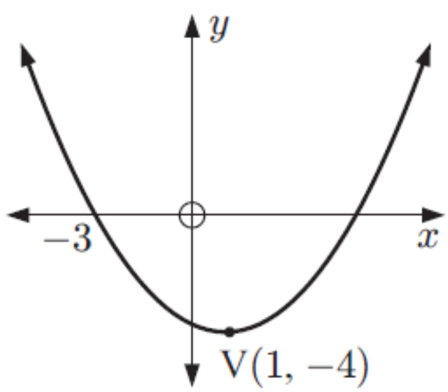
Remember the basic parts of a quadratic.

The minimum or maximum value is the vertex.
A *y*-intercept is where the *x*-value is equal to zero.
An *x*-intercept is where the *y*-value is equal to zero.

This can be helpful when trying to write an equation from given information or a given graph.

When dealing with a story problem, initial values are *y*-intercepts. And many times you are looking for minimums or maximums. This would be the vertex. And many times you are looking for places where the *y*-value is zero. This would be *x*-intercepts.

Example: Write an equation for the graph shown below.



If I have an *x*-intercept at –3, then I have a point at (–3, 0). Since my axis of symmetry would be at $x = 1$, then I must also have an *x*-intercept at 5. That means I have another point at (5, 0).

An equation for a quadratic could be write factored in the form $y = a(x - x_1)(x - x_2)$ where x_1 and x_2 are intercepts.

So in this case…
$y = a(x + 3)(x - 5)$

Now plug in my vertex as my *x* and *y* and solve for *a*…

$-4 = a(1 + 3)(1 - 5)$
$-4 = a(4)(-4)$
$-4 = -16a$
$a = 0.25$

So that makes my equation…
$\boldsymbol{y = 0.25(x + 3)(x - 5)}$

If they insist on it being in the form $ax^2 + bx + c$…
$y = 0.25(x^2 - 2x - 15)$
$\boldsymbol{y = 0.25x^2 - 0.5x - 3.75}$

Example: Andrew shoots an arrow off a jetty. The arrow's height above the water t seconds after being shot is given by $H(t) = -2t^2 + 6t + 3$ metres, $t \geq 0$.

a) How high above the water is Andrew when he shoots the arrow off the jetty?

This is the y-intercept. What is the value when $t = 0$?

3 metres

b) How high above the water is the arrow after 1 seconds?

Plug in $t = 1$

$-2(1)^2 + 6(1) + 3 = 7$ metres

c) How long does it take for the arrow to reach the maximum height?

The x-value is time. This is the x-value of the vertex. Graph it or use $\frac{-b}{2a}$

$$\frac{-b}{2a} = \frac{-6}{2(-2)} = \frac{-6}{-4} = 1.5 \text{ seconds}$$

d) How far above the water is the arrow at its highest point?

This is the y-value of the vertex. Plug in $t = 1.5$

$-2(1.5)^2 + 6(1.5) + 3 = 7.5$ metres

e) How long does it take for the arrow to hit the water?

This is when the arrow has a height of zero. That means $y = 0$. That means x-intercepts

$0 = -2t^2 + 6t + 3$

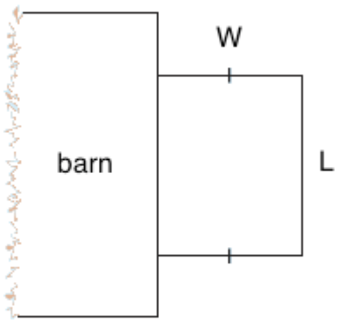
This doesn't factor so either use the Quadratic Formula, PlySmlt2 App, or graph to find the zeros...

The only appropriate answer is 3.44 seconds (3 sig figs)

f) Tyler decides to shoot his gun off the jetty in hopes of hitting Andrew's arrow. The equation that models the path of the bullet is $H(t) = t^2 + 5t - 4$. Does he hit the arrow? And if so, how long does it take to hit it?

Graph both equations and find the intersection using Calc Intersect. Since they do intersect that means the bullet does hit the arrow. Find the x-value since that is time It happens at 1.70 seconds.

Example: A farmer has 20 metres of fencing to create a rectangular enclosure next to his barn. Two sides of the enclosure are equal lengths.



a. Write a linear equation for L in terms of W.

Write an equation for the perimeter...
2W + L = 20
L = 20 – 2W

b. Write a quadratic equation for the Area in terms of W.

A = WL
A = W(20 – 2W)
A = 20W – $2W^2$
A = $-2W^2$ + 20W

c. Find the dimensions of the maximum possible enclosure.

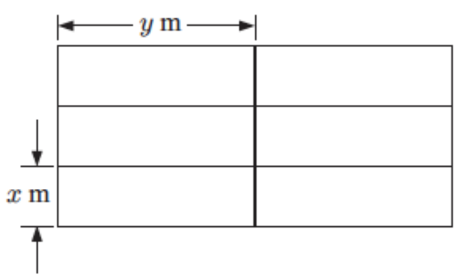
I need the maximum so find the vertex by graphing or $x = \frac{-b}{2a}$

$x = W = \frac{-20}{2(-2)} = \frac{-20}{-4} = 5 \text{ metres}$

I don't need the *y*-value right now. Find L by using part a.
L = 20 – 2W = 20 – 2(5) = 10 metres

d. Find the area of the enclosure: A = LW = (10)(5) = 50 m^2

Example: Six identical adjacent pens are fenced as shown below using 1800 metres of fencing.



a. Explain why 9*x* + 8*y* = 1800

There is 1800 metres of fencing. If I add up all the sides that are fenced I have 9 sides of x and 8 sides of y.

b. Show that the area of each pen is $A = \frac{-9}{8}x^2 + 225x$ m^2

Area = xy

Solve the equation above for *y*

8y = –9x + 1800 $\Rightarrow$ $y = \frac{-9}{8}x + 225$

Now plug that into the area formula

Area = $x\left(\frac{-9}{8}x + 225\right) = \frac{-9}{8}x^2 + 225x$

c. If the area is to be maximized, what would be the dimensions of each pen?

Find the vertex...

$x = \frac{-225}{2(-9/8)} = \frac{-225}{-2.25} = 100 \text{ metres}$

Plug that back into part a to get y.

Dimensions: 100 m by 112.5 m