Section 4.6 Quadratic Functions

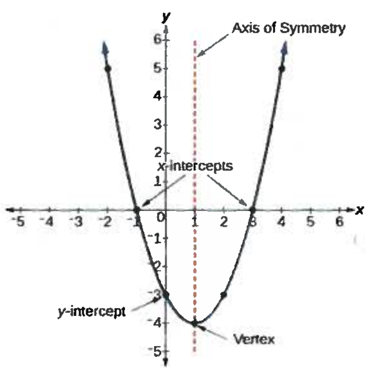
Objective:

Analyze the characteristics of graphs of quadratic functions. Graph quadratic functions.

Characteristics of Quadratic Functions:

Quadratic functions are nonlinear and can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$.

- This form is called the standard form of a quadratic function.
- The shape of the graph of a quadratic function is called a parabola.
  It is a U shape – right side up or upside down.
- Parabolas are symmetric about a central line called the axis of symmetry.
- The axis of symmetry intersects a parabola at only one point, called the vertex.
  This is the minimum or maximum value of the graph.
- There is always a y-intercept at "c" but there is not always an x-intercept.



- When $a > 0$, the graph of $y = ax^2 + bx + c$ opens upward and has a minimum.
  When $a < 0$, the graph opens downward and has a maximum.
  The maximum or minimum is the vertex.

- The x-value of the vertex (the axis of symmetry) can be found two ways.
  1) By using the graphing calculator. Go to Calc (2nd TRACE) then Min or Max
  2) By hand by using the formula $x = \frac{-b}{2a}$. You can then find the *y*-value by plugging the *x*-value back in to the original formula.

- Quadratics can be graphed using symmetry. You need the vertex and at least one point on one side. You can then duplicate that point on the other side of the axis of symmetry.

Example 1: Find the vertex, the equation of the axis of symmetry, the x-intercept (if any) and the y-intercept of each function. Then state whether the graph has a minimum or a maximum. Then draw a sketch of the graph.

a. $y = -2x^2 - 2x + 4$

$a = -2 \quad b = -2 \quad c = 4$

$\frac{-b}{2a} = \frac{-(-2)}{2(-2)} = \frac{2}{-4} = -0.5$

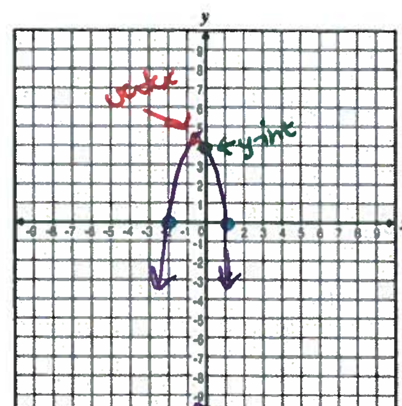
$y = -2(-0.5)^2 - 2(-0.5) + 4 = 4.5$

Since $a < 0$ ← max

y-int:
$y = -2(0)^2 - 2(0) + 4 = 4$

Vertex: $(-0.5, 4.5)$
Minimum/Maximum? max
Axis of Symmetry: $x = -0.5$
X-Intercept(s): $x = -2, x = 1$
Y-Intercept: $y = 4$

x-int:
$0 = -2x^2 - 2x + 4$
$0 = -2(x^2 + x - 2)$
$0 = -2(x+2)(x-1)$
$x = -2, x = 1$



b. $y = x^2 - 6x + 13$

$a = 1 \quad b = -6 \quad c = 13$

$\frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

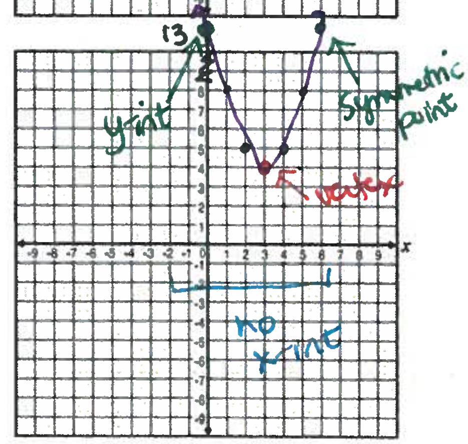
$y = (3)^2 - 6(3) + 13 = 4$

Since $a > 0$ ← min

y-int:
$y = (0)^2 - 6(0) + 13 = 13$

Vertex: $(3, 4)$
Minimum/Maximum? min
Axis of Symmetry: $x = 3$
X-Intercept(s): none
Y-Intercept: $y = 13$

x-int:
$0 = x^2 - 6x + 13$
doesn't factor!
use Quadratic Formula
or PlySmlt 2 app → no solution

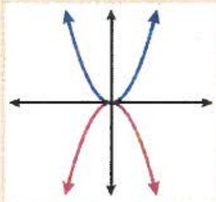


Recall that functions can also be reflected, stretched, or compressed.

(x, y)
$(x, -y)$

Reflections

Reflection Across x-axis



Output values change.

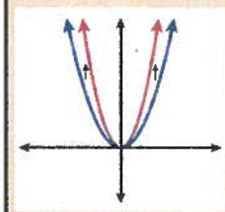
$f(x) = x^2$

$-f(x) = -(x^2)$
$= -x^2$

The function is flipped across the x-axis.

Stretches and Compressions

Vertical Stretch/Compression by a Factor of $|a|$



Output values change.

$f(x) = x^2$

$a \cdot f(x) = ax^2$

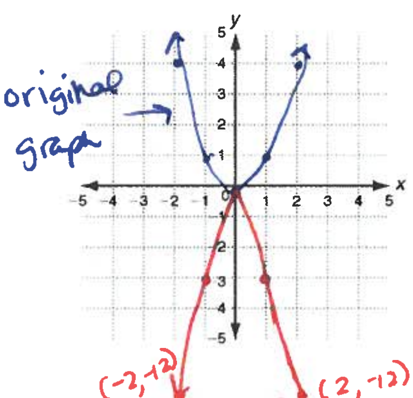
$|a| > 1$ stretches away from the x-axis.

$0 < |a| < 1$ compresses toward the x-axis.

(x, y)
(x, ay)

Example 4: **Using the graph of $f(x) = x^2$ as a guide, describe the transformations on $g(x) = -3x^2$. Then sketch the graph.**

Because a is negative, g is a reflection across the x-axis.
Because $a = 3$, g is a vertical stretch of f by a factor of 3.



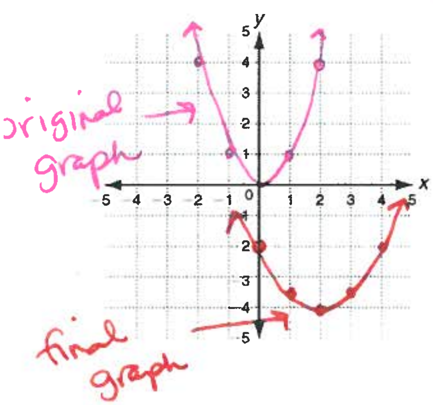
take all y's times -3 to flip + stretch
$(x, -3y)$

$(-2,4) \rightarrow (-2,-12)$
$(-1,1) \rightarrow (-1,-3)$
$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (1,-3)$
$(2,4) \rightarrow (2,-12)$

Example 5: **Use the description below to write a transformed quadratic function in vertex form. Then sketch the graph.**

The parent function $f(x) = x^2$ is vertically compressed by a factor of 1/2, ~~and then~~ translated 2 units right and 4 units down.

Always 2nd (vertically compressed by a factor of 1/2) — Always 1st (translated 2 units right) — Always Last (4 units down)



	right 2	Ver compress 1/2	Down 4
$(-2,4) \rightarrow$	$(0,4) \rightarrow$	$(0,2) \rightarrow$	$(0,-2)$
$(-1,1) \rightarrow$	$(1,1) \rightarrow$	$(1,\frac{1}{2}) \rightarrow$	$(1,-3\frac{1}{2})$
$(0,0) \rightarrow$	$(2,0) \rightarrow$	$(2,0) \rightarrow$	$(2,-4)$
$(1,1) \rightarrow$	$(3,1) \rightarrow$	$(3,\frac{1}{2}) \rightarrow$	$(3,-3\frac{1}{2})$
$(2,4) \rightarrow$	$(4,4) \rightarrow$	$(4,2) \rightarrow$	$(4,-2)$

$$\boxed{f(x) = \frac{1}{2}(x-2)^2 - 4}$$

Example 6: **Find the x and y intercept of the following: $y = -2(x-3)^2 + 8$**

X-Int: $y = 0$

$0 = -2(x-3)^2 + 8$
$-8 = -2(x-3)^2$
$4 = (x-3)^2$
$2 = x-3 \quad -2 = x-3$
$\boxed{x = 5 \quad x = 1}$

y-int: $x = 0$

$y = -2(0-3)^2 + 8$
$y = -2(9) + 8$
$\boxed{y = -10}$

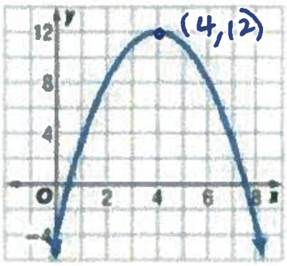
Example 2: Choose the best answer for the following.

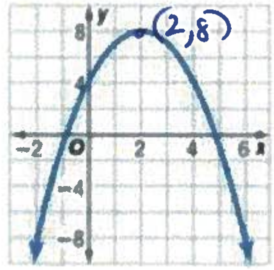
A. Ellie hit a tennis ball into the air. The path of the ball can be modeled by y = $-x^2 + 8x + 2$, where y represents the height in feet of the ball x seconds after it is hit into the air. Graph the path of the ball.

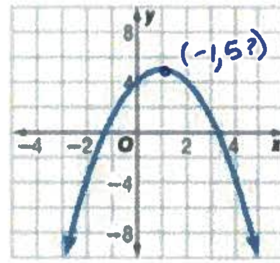
$a < 0$

$\frac{-b}{2a} = \frac{-8}{2(-1)} = 4$

$-(4)^2 + 8(4) + 2 = 18$

A.


B.


C.


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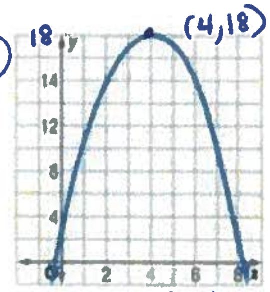


right direction
right vertex!

B. At what height was the ball hit?

let $x=0$ $y = 0^2 + 8(0) + 2 =$ 2 feet

C. What is the maximum height of the ball?

y-value of vertex → 18 feet

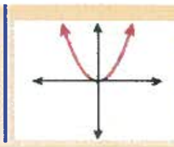
D. When did the ball hit the ground?

let $y=0$ (height $=0$) $0 = -x^2 + 8x + 2$ won't factor... use graph or app to solve

$\boxed{x = 8.2 \text{ seconds}}$

The parent function $f(x) = x^2$ has its vertex at the origin (0, 0).

x	−2	−1	0	1	2
$f(x) = x^2$	4	1	0	1	4



Transforming quadratic functions can be done. Horizontal and vertical translations change the vertex of the parent function $f(x) = x^2$. Keeping that in mind, you can graph using translations.

Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k$$

a indicates a reflection across the *x*-axis and/or a vertical stretch or compression.

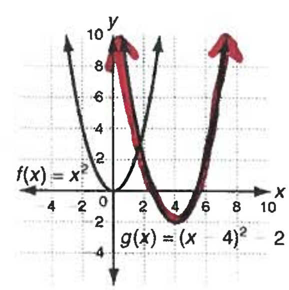
h indicates a horizontal translation.

k indicates a vertical translation.

Note: *a*, *h*, and *k* are constants.

Parent Function	Transformation
$f(x) = x^2$	$g(x) = (x - h)^2 + k$
Vertex: (0, 0)	Vertex: (h, k)

The translation shifts $f(x) = x^2$: h units right ($h > 0$) or left ($h < 0$) & k units up ($k > 0$) or down ($k < 0$).



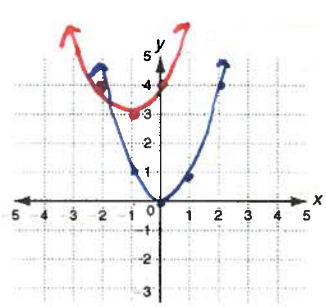
The vertex of $g(x) = (x - 4)^2 - 2$ is (4, −2).
The graph of $f(x) = x^2$ is shifted 4 units right and 2 units down.

Example 3: **Use the graph of $f(x) = x^2$ as a guide. Find the translation, new vertex, and sketch the graph**

$g(x) = (x + 1)^2 + 3$

left 1 up 3

(−1, 3)



$g(x) = x^2 - 2$

down 2

(0, −2)