IB Math Studies
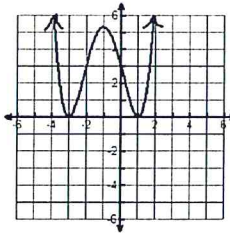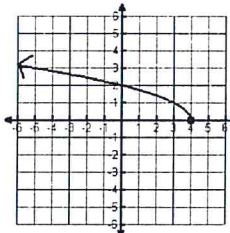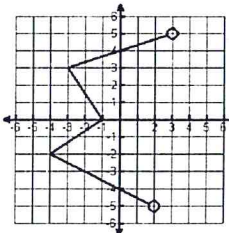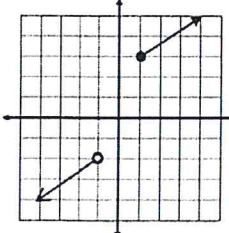## Topic 6 Day 1: Functions & Relations Sections 4.1-4.3

♦ **relation:** a set of ordered pairs
  ○ **domain (X):** the set of all first elements of a relation (the independent variable)
  ○ **range (Y):** the set of all second elements of a relation (the dependent variable)

1. a relation is defined by the rule $y = x + 2$, where $x \in \{0, 1, 2, 3, 4\}$

    (a) determine the range of the relation $-$ plug in 0,1,2,3,4

    plug in 0,1,2,3,4 ⤴
    2,3,4,5,6   So range = $\{2,3,4,5,6\}$

    (b) express the relation as a set of ordered pairs

    $(0,2) (1,3)(2,4)(3,5)(4,6)$
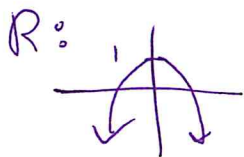
2. **Determine the domain and range of the relations**

a) Domain:	b) Domain:	c) Domain:	d) Domain:	e) Domain:
$\mathbb{R}$	$\mathbb{R}$	$(-\infty, 4]$	$[-4, 3)$	$(-\infty, -1) \cup [1, \infty)$
Range:	Range:	Range:	Range:	Range:
$\mathbb{R}$	$[0, \infty)$	$[0, \infty)$	$(-5, 5)$	$(-\infty, -2) \cup [3, \infty)$

3. Determine the domain and range of the relations ⟶ use the equations + graphs
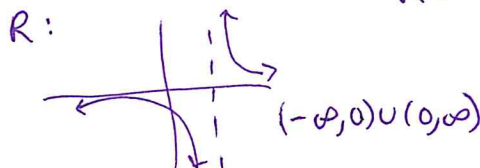
    a) $y = 1 - x^2$ b) $y = \frac{1}{x-5}$ c) $y = \sqrt{x - 5}$

    D: $\mathbb{R}$ D: Can't divide   D: can't take $\sqrt{-\#}$
                                by zero $x - 5 \geq 0$
    R: $x - 5 \neq 0$ $x \geq 5$ or $[5, \infty)$
                             $x \neq 5$
                                                         R:
    $(-\infty, 1]$ R: $(-\infty, 0) \cup (0, \infty)$ $[0, \infty)$
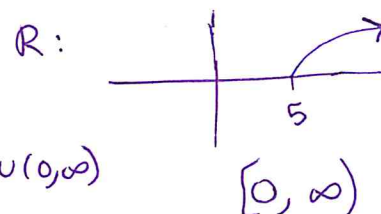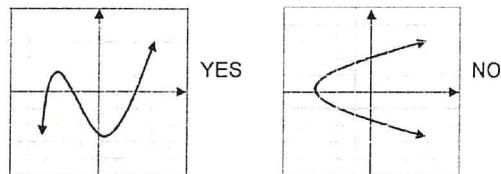
- ◆ **function:** a relation in which each element of the domain (x) is paired with only one element of the range (y). All functions are relations, but not all relations are functions

  Tests for functions:

  1. Check the ordered pairs to make sure that each input gives only one output
  2. Check to see if the graph passes the vertical line test

  YES   NO

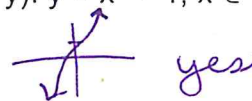 3. Put numbers in for x and check for one input = one output

4. Which of the following defines a function?

   a) {(0, 2), (1, 2), (2, 1)}

   ~~f∴~~ *yes*

   b) {(x, y): y = x³ + 1, x ∈ ℜ}

   yes

   c) $3x^2 + 2y = 5$

   ↑ *quadratic yes*

   d) {(x, y): $3x^2 + 2y^2 = 5$}

   no! can't have power on y!

Function notation

The set X is called the **domain** of f and Y is called the **range** (or **co-domain**). The element y is called the **image** of x under f and we denote this image by f(x), the value of the function f at x (read "f of x")

5. For the function $f(x) = x^2 + 2x$, x ∈ ℜ find

   a) f(1)

   $(1)^2 + 2(1)$

   3

   b) f(-3)

   $(-3)^2 + 2(-3)$

   $9 - 6$

   3

   c) f(a + b)

   $(a+b)^2 + 2(a+b)$

   $a^2 + 2ab + b^2 + 2a + 2b$

Composition Functions

The expression $f(g(x))$ or $(f \circ g)(x)$ is called a composition function. In both cases you plug the input of $g(x)$ into $f(x)$.

6. Example: Let $f(x) = 3x - 2$ and $g(x) = x^2$. Find...

   a) $g(f(-2))$

   $f(-2) = 3(-2) - 2$

   $f(-2) = -8$

   $g(-8) = (-8)^2$

   $\boxed{= 64}$

   b) $(f \circ g)(3) = f(g(3))$

   $g(3) = 3^2 =$

   $g(3) = 9$

   $f(9) = 3(9) - 2 = \boxed{25}$