Section 2.7 Quadratic Equations

A quadratic equation is an equation that can be written in the form $ax^2 + bx + c = 0$. It's possible that b and/or c could be zero. In general, a quadratic is solved either by factoring or by using the quadratic formula.

If my equation doesn't have a b value, try factoring using difference of squares or square roots.
Example: Find the roots (zeros) of each function.

a) $0 = x^2 - 64$

$a = x \quad b = 8$

$0 = (x + 8)(x - 8)$

$x = -8$ and 8

b) Alternately...
Solve by Square Root

$0 = x^2 - 64$

$x^2 = 64$

$x = \pm 8$

c) $4x^2 = 49$

$4x^2 - 49 = 0$

$a = 2x \quad b = 7$

$(2a + 7)(2a - 7) = 0$

$x = -7/2$ and $7/2$

Using a chart when *a* = 1: You have a trinomial of the form $ax^2 + bx + c$. Look for factors of c that will add to give you b. Always look to see if you need to pull out a GCF before starting…

Example: Solve $x^2 - 11x + 24 = 0$

Factors of 24	Sum (–11)
– 2, – 12	– 14
– 4, – 6	– 10
–3, – 8	– 11

Since –3 and – 8 both multiply to give me 24 and add to give me –11, those are my factors…

$(x - 3)(x - 8) = 0$ Solutions: 3 and 8

Example: Solve $2x^2 + 8x - 10 = 0$.

First factor a GCF of 2… $2(x^2 + 4x - 5) = 0$

Factors of – 5	Sum (4)
5, – 1	4

$2(x + 5)(x - 1) = 0$ Solutions: –5 and 1

Note: if your GCF is a number, don't worry about setting it equal to zero. You only have to do that with factors that have an x in them.

Example: Solve $x^2 + 3x = 10$.

This would be $x^2 + 3x - 10 = 0$

Factors of (– 10)	Sum (3)
– 5, 2	– 3
5, – 2	3

$(x + 5)(x - 2) = 0$ Solutions: –5 and 2

Example: Solve $x^2 = 7x$

This would be $x^2 - 7x = 0$

First factor a GCF of x…

$x(x - 7) = 0$

$x = 0 \qquad x - 7 = 0$
$\qquad\qquad x = 7$ Solutions: 0 and 7

Factoring when $a \neq 1$: You have a trinomial of the form $ax^2 + bx + c$. You looked for a GCF before starting… and didn't have one.

- Method 1: Look for factors of ac that add to give you b.
- Rewrite your original equation and replace b with those two factors.
- Group and find your GCF…

Example: Solve $2x^2 - 7x + 6 = 0$.

Multiply a and c to get 12…

Factors of 12	Sum (– 7)
– 2, – 6	– 8
– 3, – 4	– 7

Now rewrite the equation and replace $-7x$ with $-3x + -4x$…

$2x^2 - 3x + -4x + 6$	Replace
$(2x^2 - 3x) + (-4x + 6)$	Group
$x(2x - 3) + -2(2x - 3)$	GCF

Remember… you want the two binomials in parenthesis to be the same… that's why we factored out a -2 in the second set of parenthesis…

$(x - 2)(2x - 3)$ So $x = 2$ and $3/2$

Example: Solve $2x^2 - 5x + 3 = 0$.

In this case we could do it the same way as the last example but let's just try to guess and check instead. Think of possible numbers that I could multiply that would work. Since my c value is positive but my b value is negative, I am looking for something like this...

(___ – ___) (___ – ___)

Now I know the only way to get a $2x^2$ is by taking $2x$ times x $\Rightarrow$ ($2x$ – ___) (x – ___)

Now the only way to get a positive c and a negative b would be with either
-1 and -3 or -3 and -1. Try them both...

$(2x - 1)(x - 3) = 2x^2 - 6x - 1x + 3 = 2x^2 - 7x + 3$... doesn't work!

$(2x - 3)(x - 1) = 2x^2 - 2x - 3x + 3 = 2x^2 - 5x + 3$... That works!

So $(2x - 3)(x - 1) = 0$ means... $x = 3/2$ and $x = 1$

The Quadratic Formula will **always** give an exact solution!

For $ax^2 + bx + c = 0\ldots \quad x = \dfrac{-b \pm \sqrt{b^2 + -4(a)(c)}}{2(a)}$

Example: Solve $3x^2 - 23x + 40 = 0$

$a = 3 \quad b = -23 \quad c = 40$

$$x = \frac{-b \pm \sqrt{b^2 + -4(a)(c)}}{2(a)} = \frac{-(-23) \pm \sqrt{(-23)^2 + -4(3)(40)}}{2(3)} = \frac{23 \pm \sqrt{529 + -480}}{6} = \frac{23 \pm \sqrt{49}}{6} = \frac{23 \pm 7}{6}$$

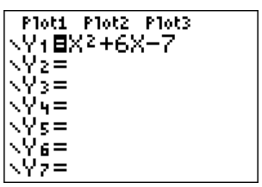
Now… this actually gives me two answers…

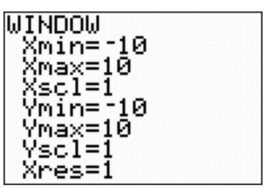
$$x = \frac{23+7}{6} = \frac{30}{6} = 5 \qquad\qquad x = \frac{23-7}{6} = \frac{16}{6} = \frac{8}{3}$$

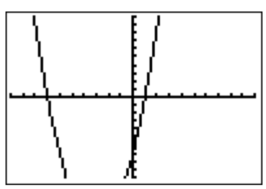
Solve using the Graphing Calculator

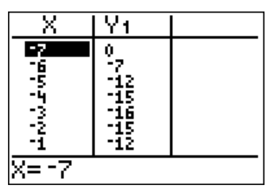
Method 1: Solve by graphing
In order to solve by graphing, you are looking for the x-intercepts (where $y = 0$). Use your table to find these or if they are a decimal, use CALC (2nd TRACE) and ZERO to find them.

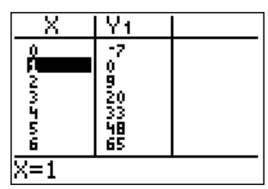
Example: Solve $f(x) = x^2 + 6x - 7$ by graphing.



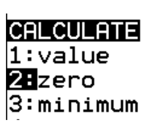


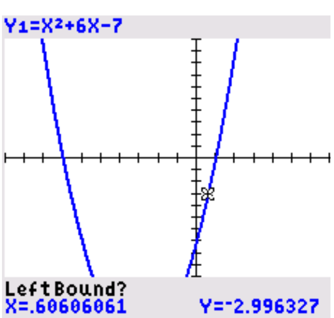


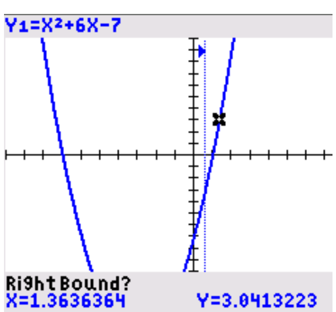


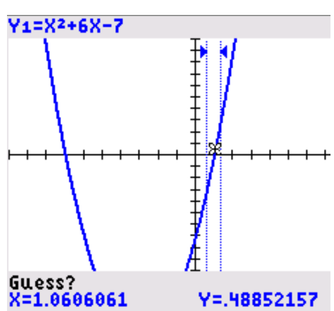


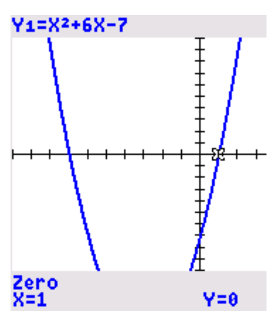
If using CALC/ZERO...







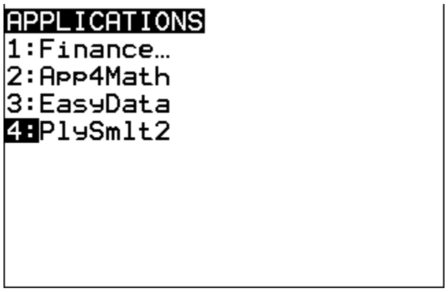


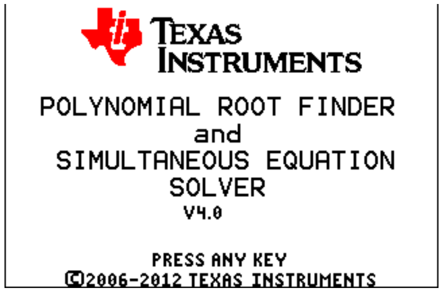


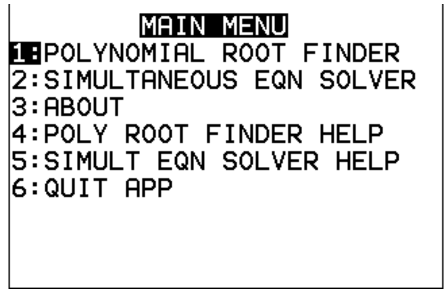
Answer: – 7 and 1

Method 2: Use the Polysmlt2 App

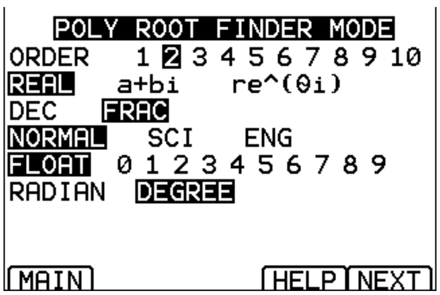
Solve $3x^2 - 23x + 40 = 0$

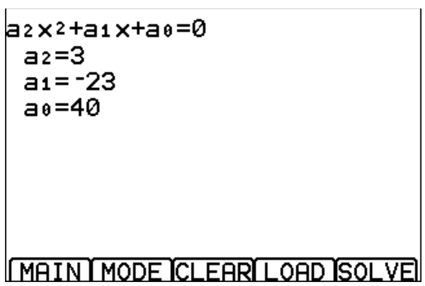


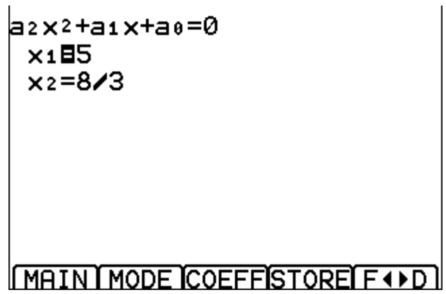




Screenshots on a TI-84 Plus or Plus C

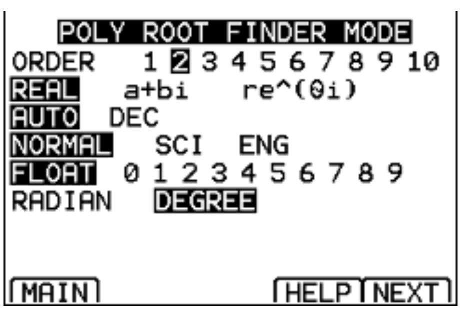


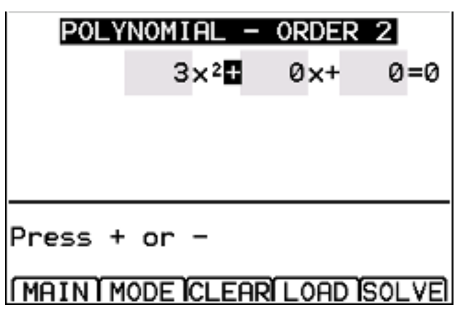


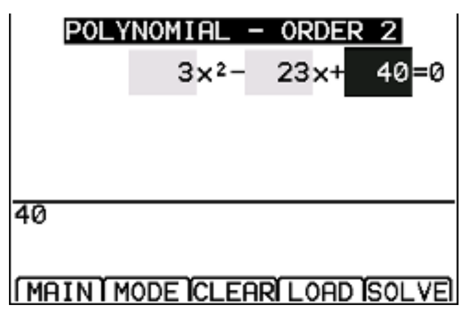


Solutions: $x = 5$ or ${}^{8}/_{3}$

Or if you have a TI-84 Plus CE the screens look a little different... choose Auto for it to give you a fraction (if applicable) and type in your coefficients. Be sure to change the operation sign to a subtraction if the coefficient is negative and use a plus sign if it is positive.







The bad thing about these calculator methods is that they may not give you an exact answer. If it doesn't say you can't use the calculator to solve and it asks for an exact answer and the calculator won't give you one then you may have to use another method. Remember... the quadratic formula, while more complicated, will always work and will always give you an exact answer.