

## Section 2.2 Numbers in Calculations

hundreds  
tens  
units  
tenths  
hundredths  
thousandths

**203.667**

one decimal place  
two decimal places  
three decimal places

Decimal notation allows us to write numbers to a chosen degree of accuracy. Sometimes we want to limit our accuracy to which we write our answer, because of limitations with our original data. For instance, if we measure a table length as 185 centimeters, any calculations we use that number in are limited to three digits, or significant figures, of accuracy.

When finding significant figures, leading zeros after a decimal point or trailing zeros of numbers with no decimal place are not considered significant figures.

When giving answers, if asked to give 2 dp that means two decimal places. If asked to give 2 sf that means 2 significant figures.

### Examples:

a) 0.00023

Since the digits before the 2 and 3 are only leading zeros, the only significant digits are 2 and 3. This number has 2 sig figs.

b) 0.0050400

The significant digits are 5, 0, 4, 0, and 0. The number has 5 sig figs.

c) 10200

Since there is a 0 in between two significant digits, it counts as a significant digit. The zeros at the end do not count. The significant digits are 1, 0, and 2. This number has 3 sig figs.

d) 1252000

The significant digits are 1, 2, 5, and 2. The number has 4 sig figs.

e) 202.0030

In this number we have digits before and after the decimal point. This means that there are no leading zeroes. All the digits in this number are significant. The number has 7 sig figs.

**Example:** Calculate  $2.5(6.42^2 - 3.54)$ , giving the answer...

(a) exactly

94.191

(b) correct to two dp

94.19

(c) correct to 3 sf

94.2

## Accuracy and Range of Values

A measurement is only accurate to  $\pm 1/2$  of the smallest division on the scale.

**Example:** Rod uses a tape measure with graduations in cm to measure his height. What is Rod's height range if he estimates his height to be 188 cm?

In this case the tape measure is only accurate to  $\pm 1/2$  cm. Thus the range of heights are  $188 \pm 1/2$  cm. That can be expressed as  $187.5 \leq h < 188.5$

Note: The upper range does not include 188.5 because if it did it would round to 189 cm.

To find the error  $\varepsilon$  in a measurement or calculation use the formula

$$\varepsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\%, \text{ where } v_E \text{ is the exact value and } v_A \text{ is the approximate value.}$$

**Example:** A piece of ground in the shape of a rectangle is measured to the nearest metre. Its length is 12 m and its width is 6 m.

(a) Write the range of possible values for the length of the rectangle in the form  $a \leq \text{length} < b$ .

$$11.5 \leq l < 12.5$$

(b) Write the range of possible values for the width of the rectangle in the form  $c \leq \text{width} < d$ .

$$5.5 \leq w < 6.5$$

(c) Calculate the minimum possible perimeter of the rectangle.

$$11.5 + 11.5 + 5.5 + 5.5 = 34$$

(d) Assuming the rectangle had the minimum possible perimeter, find the error in using the given measurements as is and getting a perimeter of 36.

Let  $v_E = 34$  cm and let  $v_A = 36$  cm.

$$\varepsilon = \left| \frac{v_A - v_E}{v_E} \right| \times 100\% = \left| \frac{36 - 34}{34} \right| \times 100\% = \left| \frac{2}{34} \right| \times 100\% = .0588 \dots \times 100\% = 5.88\%$$