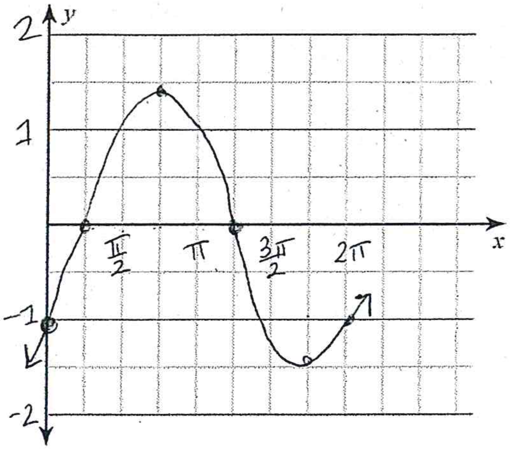
IB Math SL Section 14.2 Curve Sketching with Trigonometric Functions

Sketch each of the following functions by finding and plotting the following: *x* and *y* intercepts, relative minima and maxima, and inflection points. (unless otherwise stated).

1) $y = \sin x - \cos x$



x-int

$0 = \sin x - \cos x$

$\cos x = \sin x$

S A / T C $\quad \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

or

$1 = \frac{\sin x}{\cos x}$

$1 = \tan x$

Ref π/4 S A / T C

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

y-int

$y = \sin 0 - \cos 0$

$y = 0 - 1$

$\boxed{y = -1}$

$y' = \cos x - (-\sin x)$

$y' = \cos x + \sin x$

$0 = \cos x + \sin x$

$-\cos x = \sin x$

S A / T C $\quad \boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$

or $\quad -1 = \frac{\sin x}{\cos x}$

$-1 = \tan x$

S A / T C $\quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$

max min

+ − +

0, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π, $\frac{7\pi}{4}$, $\frac{11\pi}{6}$, 2π

$f'(\frac{\pi}{2}) = \cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$

$f'(\pi) = \cos\pi + \sin\pi = -1 + 0 = -1$

$f'(\frac{11\pi}{6}) = \cos\frac{11\pi}{6} + \sin\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2} = +$

$y'' = -\sin x + \cos x$

$0 = -\sin x + \cos x$

$\sin x = \cos x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ (see above why)

+ − +

0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, π, $\frac{5\pi}{4}$, $\frac{3\pi}{2}$, 2π

$f''(\frac{\pi}{6}) = -\sin\frac{\pi}{6} + \cos\frac{\pi}{6} = -\frac{1}{2} + \frac{\sqrt{3}}{2} = +$

$f''(\pi) = -\sin\pi + \cos\pi = 0 + -1 = -$

$f''(3\pi/2) = -\sin 3\pi/2 + \cos\frac{3\pi}{2} = -(-1) + 0 = +$

CC ↑
(0, π/4)
(5π/4, 2π)
CC ↓
(π/4, 5π/4)

Inflection Pts

$f(\pi/4) = \sin\pi/4 - \cos\pi/4 = 0 \quad (\frac{\pi}{4}, 0)$

$f(5\pi/4) = \sin 5\pi/4 - \cos 5\pi/4 = 0 \quad (5\pi/4, 0)$

inc $(0, 3\pi/4)$ $(7\pi/4, 2\pi)$ dec $(3\pi/4, 7\pi/4)$

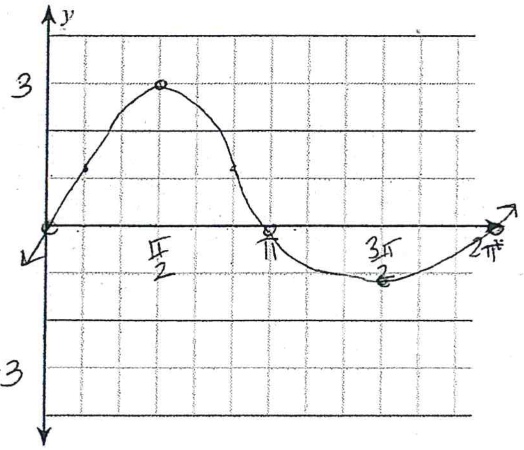
max $f(\frac{3\pi}{4}) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

$\boxed{(3\pi/4, \sqrt{2})}$

min $f(\frac{7\pi}{4}) = \sin\frac{7\pi}{4} - \cos\frac{7\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

$\boxed{(7\pi/4, -\sqrt{2})}$

2) $y = 2\sin x + \sin^2 x$



x-int

$0 = 2\sin x + \sin^2 x$

$0 = \sin x(2 + \sin x)$

$\sin x = 0 \qquad \sin x + 2 = 0$

$\boxed{x = 0, \pi, 2\pi} \qquad \sin x = -2$

no solution

y-int

$2\sin 0 + (\sin 0)^2 = y$

$2(0) + 0^2 = y$

$\boxed{y = 0}$

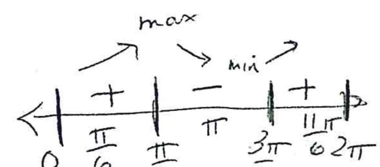
$y' = 2\cos x + \underline{2\sin x \cdot \cos x}$ (chain rule!)

$0 = 2\cos x(1 + \sin x)$

$2\cos x = 0 \qquad 1 + \sin x = 0$

$\cos x = 0 \qquad \sin x = -1$

$x = \pi/2, 3\pi/2 \qquad x = \frac{3\pi}{2}$



$f'(\pi/6) = 2\cos\pi/6 + 2\sin\pi/6\cos\pi/6 = +$

$f'(\pi) = 2\cos\pi + 2\sin\pi\cos\pi$

$2(-1) + 2(0)(-1) = -2$

$f'(\frac{11\pi}{6}) = 2\cos\frac{11\pi}{6} + 2\sin\frac{11\pi}{6}\cos\frac{11\pi}{6}$

$2(\sqrt{3}/2) + 2(-1/2)(\sqrt{3}/2)$

$\sqrt{3} - \sqrt{3}/2 = +$

max

$f(\frac{\pi}{2}) = 2\sin\pi/2 + (\sin\pi/2)^2$

$= 2(1) + (1)^2 = 3$

$\boxed{\text{max } (\pi/2, 3)}$

min

$f(3\pi/2) = 2\sin\frac{3\pi}{2} + (\sin\frac{3\pi}{2})^2$

$= 2(-1) + (-1)^2 = -1$

$\boxed{\text{min } (3\pi/2, -1)}$

inc $(0, \pi/2)(\frac{3\pi}{2}, 2\pi)$ dec $(\pi/2, 3\pi/2)$

$y'' = -2\sin x + 2(\cos x \cdot \cos x + -\sin x \cdot \sin x)$ (chain rule)

$= -2\sin x + 2(\cos^2 x - \sin^2 x)$

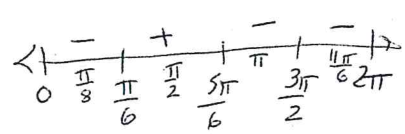
Replace $\cos^2 x$ with $1 - \sin^2 x$

$= -2\sin x + 2(1 - \sin^2 x - \sin^2 x)$

(Remember $\sin^2 + \cos^2 = 1$)

$= -2\sin x + 2(1 - 2\sin^2 x)$

$0 = -2\sin x + 2 - 4\sin^2 x$



$4\sin^2 x + 2\sin x - 2 = 0$

$2(2\sin^2 x + \sin x - 1) = 0$

$2(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = 1/2 \qquad \sin x = -1$

$x = \pi/6, 5\pi/6 \qquad x = 3\pi/2$

use calc...

$f'(\pi/8) = -$

$f'(\frac{\pi}{2}) = +$

etc.

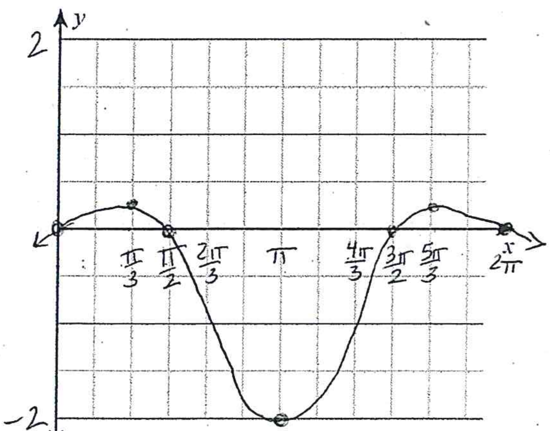
inflection points

$f(\pi/6) = 1.25 \quad \boxed{(\pi/6, 1.25)}$

$f(5\pi/6) = 1.25 \quad \boxed{(5\pi/6, 1.25)}$

CC↑ $(\pi/6, 5\pi/6)$ CC↓ $(0, \pi/6)(\frac{5\pi}{6}, 2\pi)$

3) $y = \cos x - \cos^2 x$ (Omit inflection points on this one)



x-int

$0 = \cos x - \cos^2 x$

$0 = \cos x(1 - \cos x)$

$\cos x = 0 \quad 1 - \cos x = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \quad \cos x = 1$

$\boxed{x = 0, 2\pi}$

y-int

$\cos 0 - (\cos 0)^2 = y$

$1 - 1^2 = y$

$\boxed{y = 0}$

$y' = -\sin x - 2\cos x \cdot (-\sin x)$ chain rule

$y' = -\sin x + 2\sin x \cos x$

$0 = \sin x(-1 + 2\cos x)$

$\sin x = 0 \quad 2\cos x = 1$

$x = 0, \pi \quad \cos x = 1/2$

$x = \pi/3, \frac{5\pi}{3}$

$-\ +\ -\ +\ -$: $-\frac{\pi}{6}$, 0 (min), $\frac{\pi}{6}$, $\frac{\pi}{3}$ (max), $\frac{\pi}{2}$, π (min), $\frac{3\pi}{2}$, $\frac{5\pi}{3}$ (max), 2π

$f'(-\pi/6) = -\sin\left(-\frac{\pi}{6}\right) + 2\sin\frac{-\pi}{6}\cos\frac{-\pi}{6}$

$= -(-1/2) + 2(-1/2)(\sqrt{3}/2)$

$1/2 - \sqrt{3}/2 = -$

$f'(\pi/6) = -\sin\pi/6 + 2\sin\pi/6\cos\pi/6$

$-(1/2) + 2(1/2)(\sqrt{3}/2)$

$-1/2 + \sqrt{3}/2 = +$

$f'(\pi/2) = -\sin\pi/2 + 2\sin\pi/2\cos\pi/2$

$-1 + 2(1)(0) = -$

$f'(3\pi/2) = -\sin(3\pi/2) + 2\sin(3\pi/2)\cos(3\pi/2)$

$= -(-1) + 2(-1)(0) = +$

$f'\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{11\pi}{6}\right) + 2\left(\sin\frac{11\pi}{6}\right)\left(\cos\frac{11\pi}{6}\right)$

$= -(-1/2) + 2(-1/2)(\sqrt{3}/2)$

$1/2 - \sqrt{3}/2 = -$

$f(0) = \cos 0 - (\cos 0)^2 = 1 - 1 = 0$

$\boxed{(0, 0) \text{ min}}$

$f(\pi/3) = \cos\pi/3 - (\cos\pi/3)^2$

$1/2 - 1/4 = 1/4$

$\boxed{(\pi/3, 1/4) \text{ max}}$

$f(\pi) = \cos\pi - (\cos\pi)^2$

$-1 - (-1)^2$

$-1 - 1 = -2$

$\boxed{(\pi, -2) \text{ min}}$

$f\left(\frac{5\pi}{3}\right) = \cos\frac{5\pi}{3} - \left(\cos\frac{5\pi}{3}\right)^2$

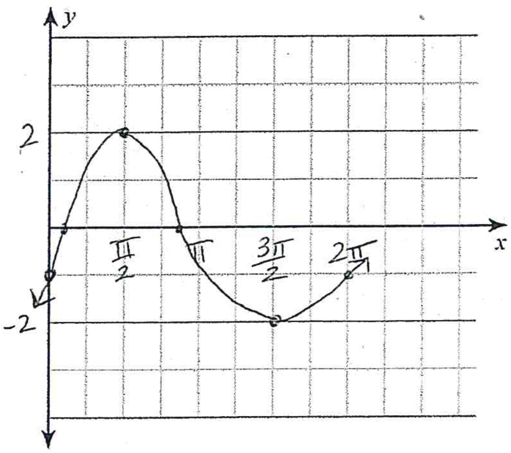
$\frac{1}{2} - \frac{1}{4} = 1/4$

$\boxed{(5\pi/3, 1/4) \text{ max}}$

inc $(0, \pi/3)(\pi, 5\pi/3)$

dec $(\pi/3, \pi)(5\pi/3, 2\pi)$

4) $y = 2\sin x - \cos^2 x$ (Omit inflection points on this one)



x int:

$0 = 2\sin x - (1 - \sin^2 x)$

$0 = 2\sin x - 1 + \sin^2 x$

$0 = \sin^2 x + 2\sin x - 1$

QF

$\sin x = .414$ $\sin x = -2.414$ no solution

$\sin^{-1}(.414)$

Ref 0.427 S A T C

Q1: $\boxed{0.427}$ Q2: π - Ref $\boxed{2.73}$

y int

$y = 2\sin 0 - (\cos 0)^2$

$y = 2(0) - (1)^2$

$\boxed{y = -1}$

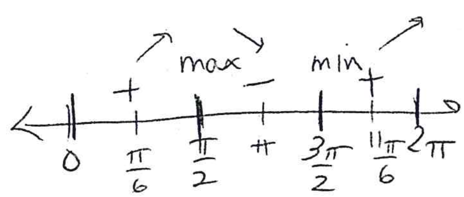
$y' = 2\cos x - \underbrace{2\cos x \cdot -\sin x}$ (chain rule)

$y' = 2\cos x + 2\sin x \cos x$

$0 = 2\cos x(1 + \sin x)$

$2\cos x = 0$ $\sin x = -1$

$x = \pi/2, 3\pi/2$ $x = 3\pi/2$



$f'(\pi/6) = 2\cos \pi/6 + 2\sin \pi/6 \cos \pi/6$

$= 2(\sqrt{3}/2) + 2(1/2)(\sqrt{3}/2) = +$

$f'(\pi) = 2\cos\pi + 2\sin\pi\cos\pi$

$= 2(-1) + 2(0)(-1) = -$

$f'(11\pi/6) = 2\cos 11\pi/6 + 2\sin \frac{11\pi}{6} \cos \frac{11\pi}{6}$

$= 2(\sqrt{3}/2) + 2(-1/2)(\sqrt{3}/2)$

$\sqrt{3} - \sqrt{3}/2 = +$

inc $(0, \pi/2)(3\pi/2, 2\pi)$
dec $(\pi/2, 3\pi/2)$

$f(\pi/2) = 2\sin(\frac{\pi}{2}) - (\cos\frac{\pi}{2})^2$

$2(1) - (0)^2 = 2$

$\boxed{(\pi/2, 2) \text{ max}}$

$f(\frac{3\pi}{2}) = 2\sin(\frac{3\pi}{2}) - (\cos\frac{3\pi}{2})^2$

$2(-1) - (0)^2 = -2$

$\boxed{(3\pi/2, -2) \text{ min}}$

$f(2\pi) = f(0) = -1$