Section 13.7 Modeling with Sine and Cosine Functions

Many real-life situations can be modeled using sine and cosine functions. Examples include tide heights, sunrise times and average temperatures.

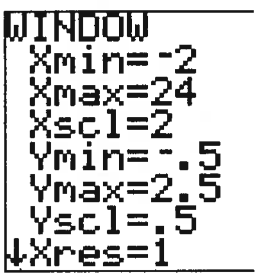
We can do this by hand by plotting data points and using the methods discussed in the last section to write a sine or cosine equation. Our graphing calculators will only do a Sine Regression (MUST BE IN RADIAN MODE). But the sine function will have the same amplitude, vertical translation, and period; its horizontal translation is one-fourth of the period to the left of the cosine curve.

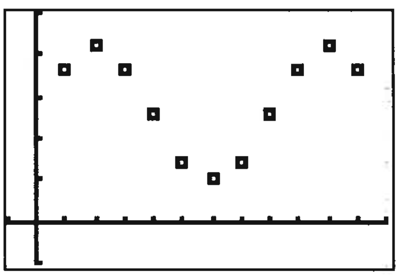
Example: The following set of data can be modeled by the function $y = A\cos(B(x \pm C)) + D$

x	2	4	6	8	10	12	14	16	18	20	22
y	1.8	2.1	1.8	1.3	0.7	0.5	0.7	1.3	1.8	2.1	1.8

a. **Use the data to estimate the period, amplitude, and vertical and horizontal transformations.**
b. **Write a cosine function to model the data**
c. **Graph the function on the same axes as the data points.**
d. **Use the regression function on your GDC to get a sine model and graph it on the same axes as the data points.**

Before doing part a, plot your points. Be in radian mode and use an appropriate window.





Amplitude: $\frac{1}{2}(2.1-0.5)=0.8$

Period: $20-4=16$

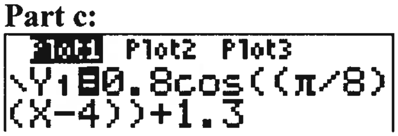
Vertical Translation: $\frac{1}{2}(2.1+0.5)=1.3$

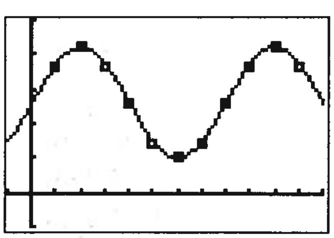
Horizontal Translation: 4 (first maximum)

Part b: Before I can write my equation, I need to find my B value.
Period = 2π/B 16 = 2π/B B = 2π/16 or B = π/8

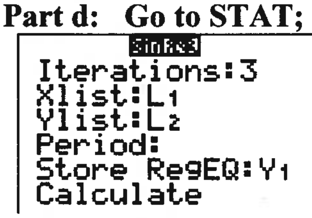
Equation: $y=0.8\cos\left(\frac{\pi}{8}(x-4)\right)+1.3$

Part c:



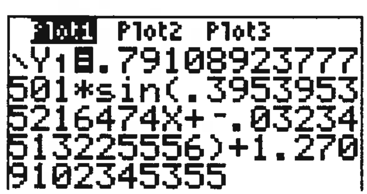


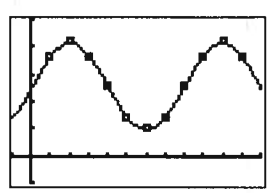
Part d: Go to STAT; CALC; SinReg



If you have the screen at the left, use the given settings

If not, use VARS to enter your regression equation into Y1





When you have a function to model data, you can use that function to make predictions.

Example: The function $h(t) = 67.5\cos\left(\frac{2\pi}{30}(t-15)\right) + 67.5$ **can be used to model the height of a passenger above the boarding platform on the London Eye.**

Part a:
Use the function to estimate the height of a passenger above the platform 8 minutes after boarding.

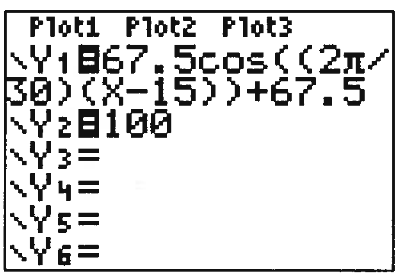
Substitute $t = 8$

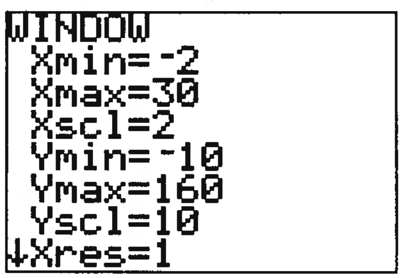
$$h(t) = 67.5\cos\left(\frac{2\pi}{30}(8-15)\right) + 67.5 \approx 74.6 \text{ metres}$$

Part b:
Use the function to estimate how long it takes for a passenger to first reach a height of 100 m.

$$67.5\cos\left(\frac{2\pi}{30}(t-15)\right) + 67.5 = 100$$

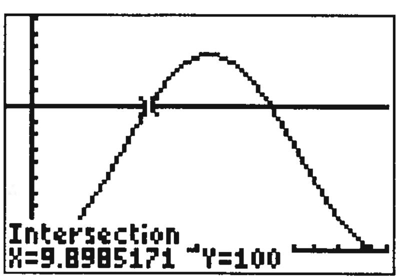
Graph both sides of the equation and use intersect to find the FIRST time $y = 100$ m





Just guess and check until you find a window that works.

Keep in mind that you need to see $y = 100$ and that you already know that $t = 8$ gives you 74.6



So $t = 9.90$ minutes

HOMEWORK L (1 & 2) & M (ALL)