Section 13.5 Translations and Stretches of Trigonometric Functions

General Equations and Transformation of Sine, Cosine, Tangent Graphs

$y = A\sin(B(x \pm C)) + D$ $\quad\quad$ $y = A\cos(B(x \pm C)) + D$ $\quad\quad$ $y = \tan(B(x \pm C)) + D$

Where:

Amplitude = A

Horizontal Translation (Phase Shift) = C

Frequency = B

Vertical Translation = D

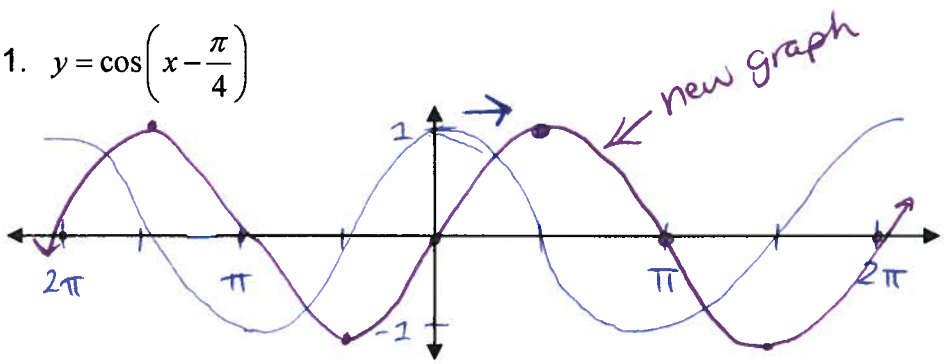
Period = 2π/B or π/B

*****Don't forget, this works the same way as other functions before with horizontal translations. So + C moves you left and – C moves you right. Also in this case a horizontal stretch or compress changes the period. If you stretch the function, the period gets larger and if you compress it the period gets smaller. If your |B| value is greater than one it will compress and if it is less than one it will stretch. A vertical stretch or compress will change the amplitude from 1 to |A|.**

In this section you should do all graphing by hand. Also, all homework problems are in radians.

Example: Graph the following for $-2\pi \le x \le 2\pi$

1. $y = \cos\left(x - \frac{\pi}{4}\right)$

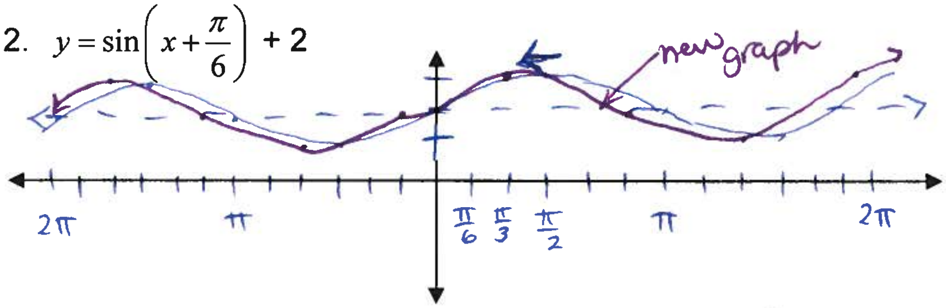


Amplitude: 1

Period: 2π

Horizontal Translation: Right π/4

2. $y = \sin\left(x + \frac{\pi}{6}\right) + 2$



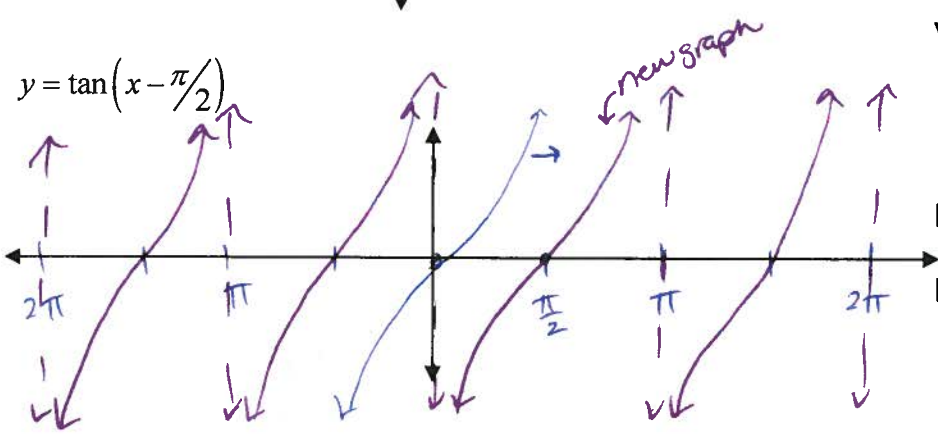
Amplitude: 1

Period: 2π

Horizontal Translation: left π/6

Vertical Translation: up 2

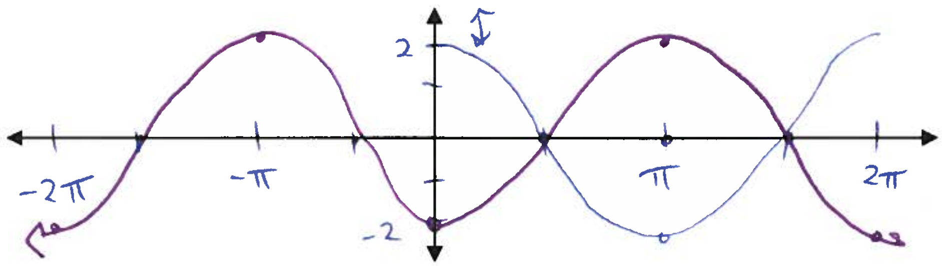
3. $y = \tan\left(x - \frac{\pi}{2}\right)$



Period: π

Horizontal Translation: right π/2

4. $y = -2\cos x$



Amplitude: 2

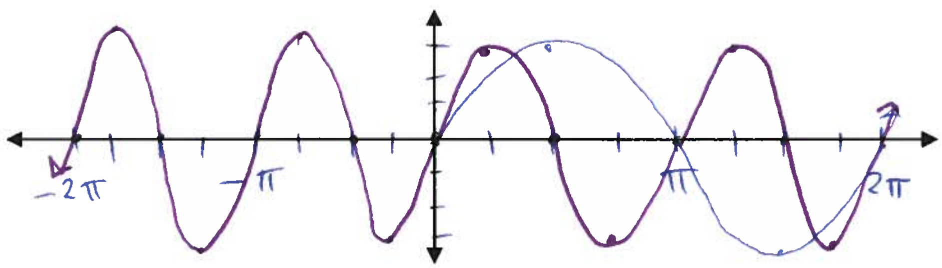
Period: 2π

{vertical reflection!}

~~Phase~~ Horizontal Shift: none

Vertical Shift: none

5. $y = 3\sin(2x)$



Amplitude: 3

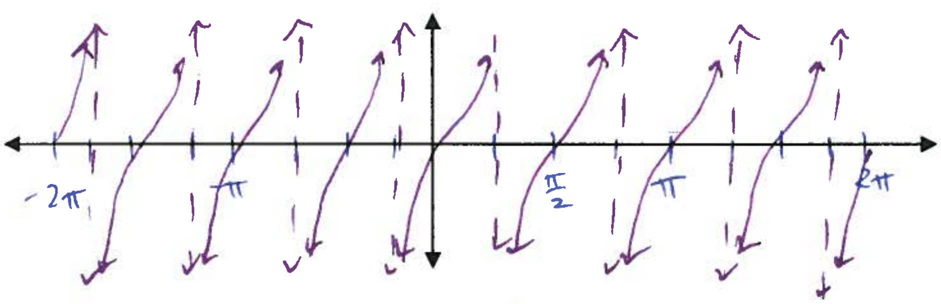
Period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

~~Phase~~ Horizontal Shift: none

Vertical Shift: none

* horizontal compress $\frac{1}{2}$

6. $y = \tan 2x$



Amplitude: —

Period: $\pi/B = \pi/2$

~~Phase~~ Horizontal Shift: none

Vertical Shift: none

* horizontal compress $\frac{1}{2}$

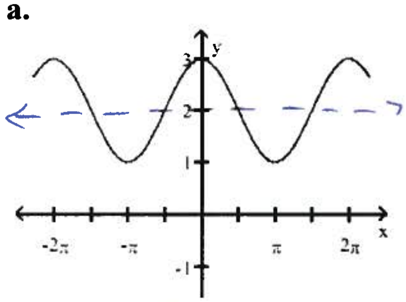
* original asymptotes at $\pm\frac{\pi}{2}$ now at $\pm\frac{1}{2}\left(\frac{\pi}{2}\right) = \pm\frac{\pi}{4}$

When writing an equation for a given graph, keep in mind what the parent function looks like

Example: Write an equation for the functions below.

* we will do this again in 13.6

a.



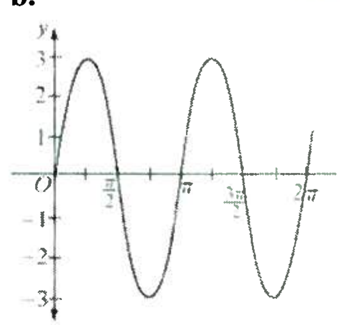
looks like cosine but moved up 2

amplitude = 1

period = 2π

$\boxed{y = \cos x + 2}$

b.

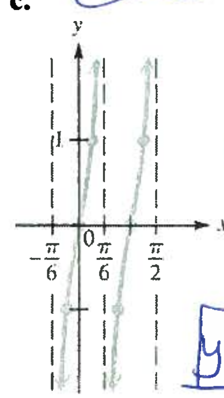


looks like sine but amplitude = 3 and compressed by $\frac{1}{2}$

(2 cycles in $[0, 2\pi]$)

$\boxed{y = 3\sin 2x}$

c.



looks like tangent compressed by $\frac{1}{3}$

(asymptotes move from $\pi/2$ to $\pi/6$)

$\boxed{y = \tan 3x}$

HOMEWORK I & J