13.5 Applications of Calculus, Part 2

Definition: Word problems involve finding the maximum or minimum values such as maximizing area or minimizing cost are known as **optimization problems**.

Tips for solving optimization problems:
1. DRAW A PICTURE!
2. Assign your variables
3. Write an equation to be **optimized** in terms of two variables
4. Find values that are *sensible* for the problem where the derivative equals zero
5. Verify that you have the desired max or min using the first or second derivative test.

*Remember to check the endpoints on a closed interval!

Example 1: The cost of making n tennis rackets each day is given by the function:
$C(n) = n^2 - 20n + 120$ where $C(n)$ is the cost in dollars per racquet.

a) How many racquets should be made per day to minimize the cost per racquet?
b) What is the minimum cost?

a) First find $C'(n)$, set it equal to 0, and solve for n.

$$C'(n) = 2n - 20$$

$$0 = 2n - 20$$

$$n = 10 \text{ racquets}$$

b) To find the minimum cost, use the value of n that you found to be the minimum number of items.

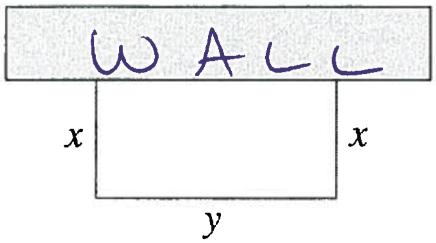
$$C(n) = n^2 - 20n + 120$$

$$C(10) = (10)^2 - 20(10) + 120$$

$$C(10) = 20$$

$20

Example 2: A rectangular plot of farmland is enclosed by 180 m or fencing material on three sides. The fourth side of the plot is bounded by a stone wall. Find the dimensions of the plot that enclose the maximum area. Find the maximum area.



180 meters of fencing = perimeter

$$x + y + x = 180$$

$$2x + y = 180$$

Maximum Area = xy

In order to maximize my area, I need only variable. So plug the perimeter equation into the area equation.

$$y = 180 - 2x$$

$$\text{Maximum Area} = xy$$

$$A = x(180 - 2x)$$

$$A = 180x - 2x^2$$

$$A' = 180 - 4x$$

$$0 = 180 - 4x$$

$$x = 45 \text{ meters}$$

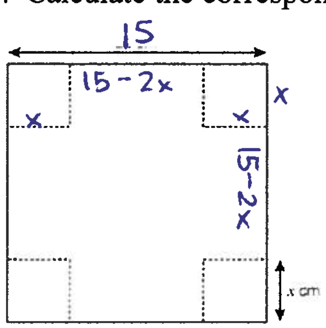
***Plug in $x = 45$ to find y

Dimensions: 45 m by 90 m

***Maximum Area

(45)(90) = 4050 m^2

Example 3: A square piece of cardboard measuring 15 cm on each side has for identical squares cut out of the corners so that it can be folded up to make a box with an open top. Determine the value of x for which the volume of the box is maximum. Hence find the dimensions of the box that give the maximum volume. Calculate the corresponding volume.



$$\text{Volume} = lwh$$

$$V = (15 - 2x)(15 - 2x)(x)$$

$$V = (225 - 60x + 4x^2)(x)$$

$$V = 225x - 60x^2 + 4x^3$$

Now simplify the volume formula so I can find the derivative and set it equal to 0...

$$V = 225x - 60x^2 + 4x^3$$

$$V' = 225 - 120x + 12x^2$$

$$0 = 225 - 120x + 12x^2$$

Use Graphing Calc App to solve... $x = 2.5$ or $x = 7.5$

Now since $x = 7.5$ gives a side length of zero, there is only $x = 2.5$ m

So my dimensions would be 10 x 10 x 2.5 with volume 250 m^3

Example 4: The function g is defined as follows:

$$g: x \mapsto px^2 + qx + c, \text{ where } p, q, c \in \mathbb{R}$$

a) If $g'(x) = -4x + 12$, find the values of p and q.

First find $g'(x)$ of the given function... remembering that p and q are numbers...

$$g'(x) = 2px + q$$

Now set it equal to the actual derivative...

$$2px + q = -4x + 12$$

So obviously $q = 12$. And since $2p = -4$ that means that $p = -2$

So that makes $g(x) = -2x^2 + 12x + c$

b) If g has a maximum value of 5 at point A, find the value of c.

If I know point A then I could plug it in to $g(x)$ and solve for c. So find the maximum.

Set $g'(x) = 0$

$-4x + 12 = 0$

$x = 3$

So that means A = (3, 5)

Now plug it in to $g(x)$ find c...

$$5 = -2(3)^2 + 12(3) + c$$

$5 = -18 + 36 + c$

$c = -13$