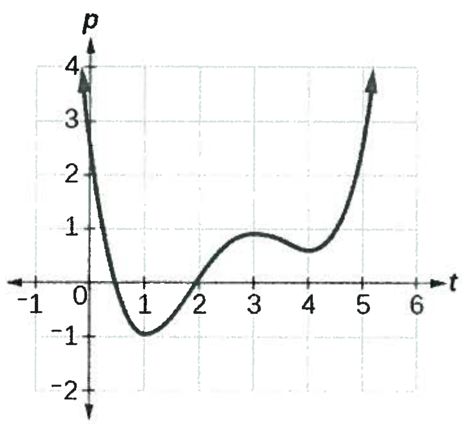
13.5 Applications of Calculus, Part 1

Definition: A function has a **local maximum** (or relative) when the function changes from increasing to decreasing. Hence, if $f'(x)$ changes from positive to negative at $x = c$, then f has a relative maximum at $(c, f(c))$.

Definition: A function has a **local minimum** (or relative) when the function changes from decreasing to increasing. Hence, if $f'(x)$ changes from negative to positive at $x = c$, then f has a relative minimum at $(c, f(c))$.

*Note: if $f'(x)$ does not have to change sign at a critical number.
If this happens, there is neither a relative maximum or minimum at that point.

Example 1: Estimate the local maxima and minima of the graph below.



local maxima (peaks)
pt (3, 1)

local minima (valleys)
pt (1, −1) pt (4, 0.5)

Example 2: Find the coordinates of the local minimum and maximum of $f(x) = 3x^4 + 4x^3 - 36x^2$

$f'(x) = 12x^3 + 12x^2 - 72x$

$0 = 12x(x^2 + x - 6)$

$0 = 12x(x+3)(x-2)$

$12x = 0$ $\quad x + 3 = 0$ $\quad x - 2 = 0$

$x = 0$ $\quad x = -3$ $\quad x = 2$

$f'(x)$

	−4		−3		−1		0		1		2		3
		− ↘		+ ↗				− ↘				+ ↗	
			min				max				min		

$f'(-4) = -288$ $\quad f(1) = -48$

$f(-1) = 72$ $\quad f(3) = 216$

$f(-3) = 3(-3)^4 + 4(-3)^3 - 36(-3)^2$

$f(-3) = -189$

$f(0) = 3(0)^4 + 4(0)^3 - 36(0)^2$

$f(0) = 0$

$f(2) = 3(2)^4 + 4(2)^3 - 36(2)^2$

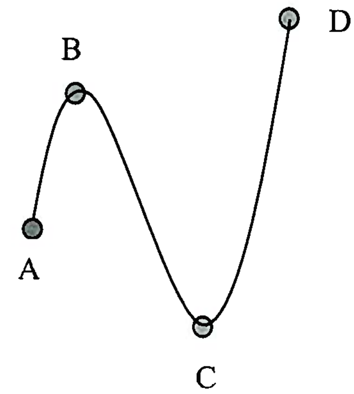
$f(2) = -64$

min: $(-3, -189)$ $(2, -64)$

max: $(0, 0)$

Definition: The **absolute (or global) extrema** of a function occur at either the local extrema or the endpoints of a function.

Example 3: Identify each labeled point as an absolute maximum or minimum, a local max or min, or neither.



Absolute max: D

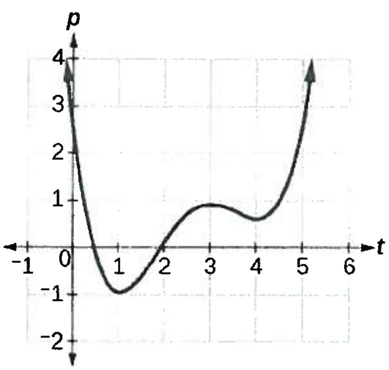
Absolute min: C

Local max (peak): B

Local min (valley): C

neither: A

Example 4: Estimate the absolute maximum and minimums of the graph on the intervals below.



Interval: $[1,4]$	Interval: $[0,4]$	Interval: $[2,5]$
Absolute Maximum:	Absolute Maximum:	Absolute Max:
$(3,1)$	$(0,2.5)$	$(5,2.5)$
Absolute Minimum:	Absolute Minimum:	Absolute Min:
$(1,-1)$	$(1,-1)$	$(2,0)$

Example 5: Find the absolute max and min for $f(x) = x^2 - 2x$ over $[-1, 2]$.

$f'(x) = 2x - 2$

$0 = 2x - 2$

$2 = 2x$

$x = 1$

$f'(x)$ $- \rightarrow +$

0 1 2

min

$f'(0) = 2(0) - 2 = -2$

$f'(2) = 2(2) - 2 = 2$

check endpoints and min

$f(-1) = (-1)^2 - 2(-1) = 3$ max

$f(2) = (2)^2 - 2(2) = 0$

$f(1) = (1)^2 - 2(1) = -1$ min

absolute max $(-1, 3)$

absolute min $(1, -1)$

Example 4: The function g is defined as follows:

$$g: x \mapsto px^2 + qx + c, \text{ where } p, q, c \in \mathbb{R}$$

a) If $g'(x) = -4x + 12$, find the values of p and q.

First find $g'(x)$ of the given function... remembering that p and q are numbers...

$$g'(x) = 2px + q$$

Now set it equal to the actual derivative...

$$2px + q = -4x + 12$$

So obviously $q = 12$. And since $2p = -4$ that means that $p = -2$

So that makes $g(x) = -2x^2 + 12x + c$

b) If g has a maximum value of 5 at point A, find the value of c.

If I know point A then I could plug it in to $g(x)$ and solve for c. So find the maximum.

Set $g'(x) = 0$

$-4x + 12 = 0$

$x = 3$

So that means A = (3, 5)

Now plug it in to $g(x)$ find c...

$$5 = -2(3)^2 + 12(3) + c$$

$5 = -18 + 36 + c$

$c = -13$