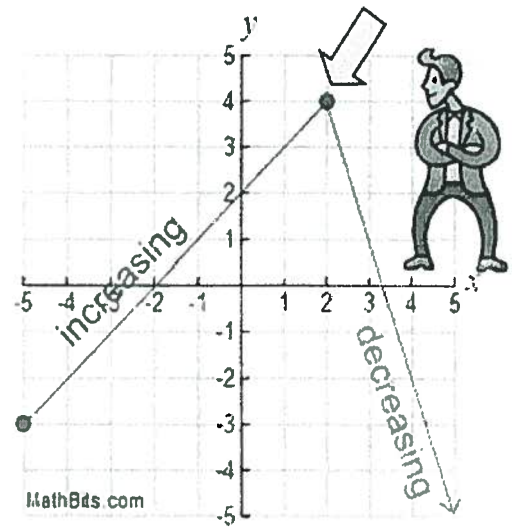
13.4 Increasing and Decreasing Functions

Definition: A function is **increasing** when the function has a positive slope.
Definition: Similarly, a function is **decreasing** when the function has a negative slope.

Differing notations for intervals:

Take a look at the point (2,4) in the graph at the right. Does that point belong to the increasing interval? The decreasing interval? Both intervals? Neither interval?



Well, the answer may be both, neither, or a combination, depending upon the convention you are following. You may see "increasing on the interval (-5,2) or the interval [-5,2], or the interval (-5,2], or the interval [-5,2). Just be consistent with the convention you are using.

Personally I like using (-5,2). Your book uses [-5,2]

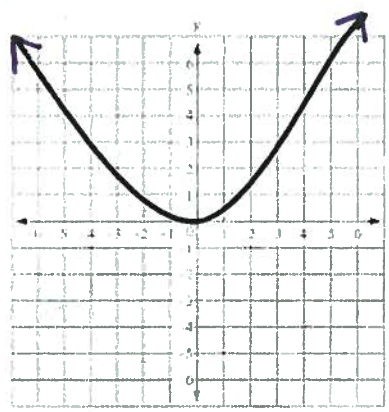
References to $\pm$ infinity, however, are always "open" notation. So the decreasing interval could only be written as $(2, \infty)$ or $[2, \infty)$

NEVER use a bracket with infinity!!!

Definition: If $f'(x) > 0$ (positive) for all x in interval (a,b), then f is **increasing** on (a,b).
Definition: If $f'(x) < 0$ (negative) for all x in interval for all x in (a,b), then f is **decreasing** on (a,b).
Definition: A **stationary point** is where $f'(x) = 0$. On the graph this represents a minimum or maximum point.

Example 1: Write the intervals on which the function is increasing and decreasing.

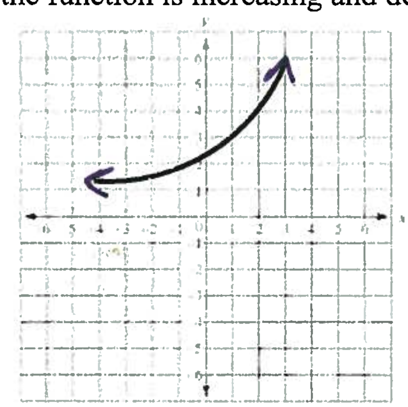
a.



Inc: $(0, \infty)$

Dec: $(-\infty, 0)$

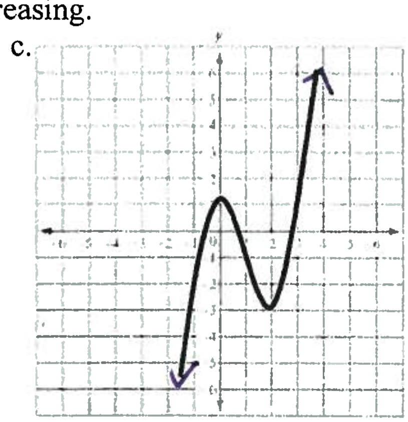
b.



Inc: $(-\infty, \infty)$

Dec: none

c.



Inc: $(-\infty, 0) \cup (2, \infty)$

Dec: $(0, 2)$

Remember... intervals of x-values!

Example 2: Use the derivative of f to find the intervals on which f is increasing or decreasing.

a.) $f(x) = -3x^2 - 18x$

$f'(x) = -6x - 18$

$0 = -6x - 18$

$18 = -6x$

$x = -3$

$f'(x)$

$-6 \quad -3 \quad 0$

$f'(-6) = 18$ $\quad f'(0) = -18$

$+ \quad -$

$-6 \quad -3 \quad 0$

inc $(-\infty, -3)$

dec $(-3, \infty)$

b.) $f(x) = 2x - 7$

$f'(x) = 2$

$0 \neq 2$

So since $f'(x) = 2$ all the time $f'(x)$ is positive all the time

inc $(-\infty, \infty)$

c.) $f(x) = 2x^3 - 3x^2 - 12x$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6x^2 - 6x - 12$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2 \quad x = -1$

$f'(x)$

$-2 \quad -1 \quad 0 \quad 2 \quad 3$

$f'(-2) = 24 \quad f'(0) = -12$

$f'(3) = 24$

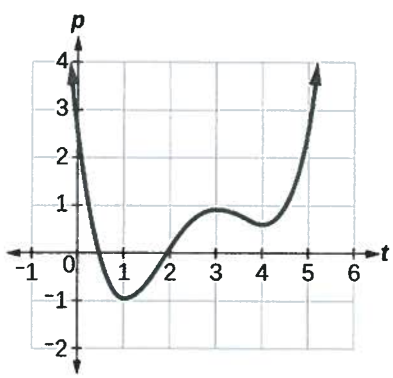
$+ \quad - \quad +$

$-1 \quad 2$

inc $(-\infty, -1)$ $(2, \infty)$

dec $(-1, 2)$

Example 3: Use the graph of $f(x)$ below, to answer the following questions



a) Where is $f(x)$ increasing? Where is $f(x)$ increasing the most?

$(1, 3)$ $(4, \infty)$ $\quad$ steepest : $(4, \infty)$

b) Where is $f(x)$ decreasing? Where is $f(x)$ decreasing the most?

$(-\infty, 1)$ $(3, 4)$ $\quad$ steepest : $(-\infty, 1)$

c) Where is $f'(x) = 0$?

slope of zero $\Rightarrow$ $x = 1, x = 3, x = 4$

d) Where does the graph have tangent lines with positive slopes?

Same as Part a $\Rightarrow$ $(1, 3)$ $(4, \infty)$

e) Where does the graph have tangent lines with negative slopes?

Same as Part b $\Rightarrow$ $(-\infty, 1)$ $(3, 4)$

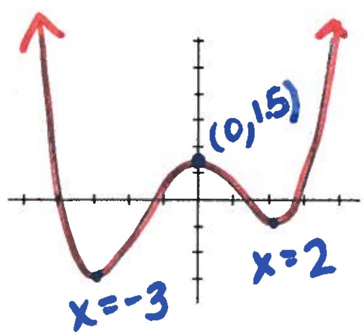
f) Where does the graph have horizontal tangent lines?

Same as Part c $\Rightarrow$ $x = 1, x = 3, x = 4$

Example 4: Sketch a possible curve given the information below. Then state the value(s) of x for which your function has a horizontal derivative.

Decreasing $(-\infty, -3)$, Increasing $(-3, 0)$, Decreasing $(0, 2)$, Increasing $(2, \infty)$, $f(0) = 1.5$

Possible sketch...



Note: How low the graph goes is just a guess. It's also a guess where the graph crosses the x-axis.

That's why it says possible curve.

How high could also be a guess but in this case I was given $f(0)$ and that was a maximum.

Horizontal Derivatives for MY possible graph:

At $x = -3$ At $x = 0$ At $x = 2$

Example 5: If a function is of the form $y = ax^2 + 5x - 4$ and $f(-3) = -1$, solve for a.

If $f(-3) = -1$, that means $(-3, -1)$

$y = ax^2 + 5x - 4$

$-1 = a(-3)^2 + 5(-3) - 4$

$-1 = 9a - 15 - 4$

$-1 = 9a - 19$

$18 = 2a$

$a = 9$