Section 13.3 Trigonometry Identities

An identity is true for ALL values of x. You already know two…

$$\sin^2 x + \cos^2 x = 1 \qquad\qquad \tan x = \frac{\sin x}{\cos x}$$

Page 457 and 458 show proofs of where the following identities come from

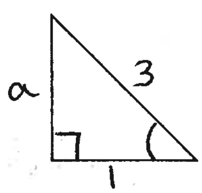
Double Angle Formulas	
$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ or $1 - 2\sin^2 x$

You can use any of the three versions of cos 2*x*.
(The last two come from substituting in the Pythagorean Identity)

S | A
T | C

Ex: Find sin2*x*, cos2*x*, tan2*x* under the condition cos *x* = – 1/3 for $\pi < x < \frac{3\pi}{2}$.

$\cos = \frac{\text{Adj}}{\text{hyp}} = \frac{1}{3}$



$1^2 + a^2 = 3^2$

$a = \sqrt{8} = 2\sqrt{2}$

By using the Pythag Thm (or the Pythag Identity), the triangle, the trig ratios, and the quadrant $\pi < x < \frac{3\pi}{2}$ that makes $\sin x = -\frac{2\sqrt{2}}{3}$.

Pythag Identity
$\sin^2 x + (-1/3)^2 = 1$
$\sin x = \frac{-2\sqrt{2}}{3}$

Now substitute…

$$\sin 2x = 2\sin x\cos x = 2\left(-\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right) = \frac{4\sqrt{2}}{9}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(-\frac{1}{3}\right)^2 - \left(-\frac{2\sqrt{2}}{3}\right)^2 = \frac{1}{9} - \frac{4\cdot 2}{9} = \frac{1}{9} - \frac{8}{9} = -\frac{7}{9}$$

$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{4\sqrt{2}}{9} \div -\frac{7}{9} = \left(\frac{4\sqrt{2}}{9}\right)\left(-\frac{9}{7}\right) = -\frac{4\sqrt{2}}{7}$$

Homework E

You can also use identities to help solve equations.

Example: Solve the following equations for the given domain

a. $\sin 2x - \cos x = 0$ **for** $0 \le x \le 2\pi$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ and $2\sin x - 1 = 0$

$\cos x = 0$ and $\sin x = \frac{1}{2}$

Ref $\pi/6$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ and $x = \frac{\pi}{6}, \frac{5\pi}{6}$

b. $\cos 2x = \cos x$ $\quad 0° \le x \le 180°$

$\cos 2x - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ and $\cos x = 1$

Ref $60°$

$x = 120°$ and $x = 0°$

***** You may have to substitute in an equation or a proof with Tangent or the Pythagorean Identity.**

Example: Prove the following

a. $(\sin x + \cos x)^2 \quad - \quad \sin 2x = 1$

$\sin^2 + 2\sin\cos + \cos^2 \quad - \quad 2\sin\cos = 1$

$\sin^2 + \cos^2 = 1$

$1 = 1$

b. $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

$$\cos 2x = \frac{1 - \dfrac{\sin^2 x}{\cos^2 x}}{1 + \dfrac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos 2x$

Homework F