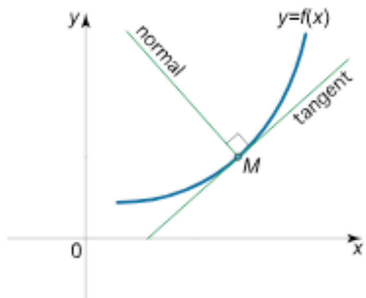
13.3 *Gradients of Curves and Equations of Tangents and Normals*

Equations of Tangent and Normal Lines



Definition: A **tangent line** is a line that touches a curve at a single point on a curve. The slope of the tangent line at a given x-value can be found by finding the value of the derivative of the function at that given x-value.

Definition: A **normal line** is a line at a point on a curve that is perpendicular to the tangent line at that point. The slope of the normal line at a given x-value can be found by finding the opposite reciprocal value of the derivative of the function at that given x-value.

Example 1: Find the gradient of the tangent and normal lines of $f(x) = 2x^2 - 5x + 1$ at $x = 3$

First find the derivative... $f'(x) = 4x - 5$

Gradient of Tangent Line at $x = 3$	Gradient of Normal Line at $x = 3$
$f'(3) = 4(3) - 5 = 7$	Opposite Reciprocal of Tangent Line: $-{}^{1}/_{7}$

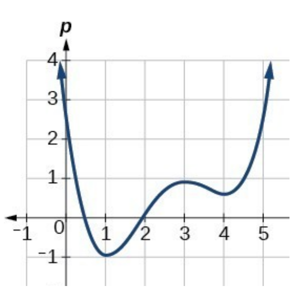
Example 2: Find the value(s) of x that give a horizontal tangent line to $f(x) = 2x^2 - 5x + 1$

A horizontal line has a slope of 0. The slope (gradient) is the value of the derivative at a particular x-value. So we need to find the x-value(s) that gives the derivative a value of 0.

$f'(x) = 4x - 5 \quad \Rightarrow \quad 4x - 5 = 0 \quad \Rightarrow \quad x = 5/4$

Example 3: State whether the functions below are increasing (positive slope) or decreasing (negative slope) at $x = 2$

a)



At $x = 2$ the function has a positive slope.

So the function is increasing at $x = 2$.

b) $f(x) = -2x^2 - 3x - 5$

In this case find the derivative.
Then check the sign of the derivative at $x = 2$.

$f'(x) = -4x - 3$

$f'(2) = -4(2) - 3 = -11$

So since $f'(2)$ is negative, the function is decreasing at $x = 2$.

Example 4: Write an equation for each line. Then graph the line and original function on the same grid.

a.) The tangent line to the curve $f(x) = x^2 + 3$ at $(1, 4)$.

First find the derivative. Then find $f'(1)$. This will be the slope of the tangent line.

$$f'(x) = 2x \quad \Rightarrow \quad f'(1) = 2(1) = 2 \quad \Rightarrow \quad \text{so } m = 2$$

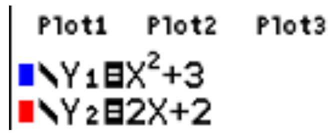
Now write the equation using your slope and the given point

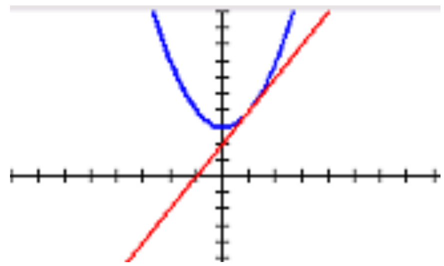
$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 1)$$

$$y - 4 = 2x - 2$$

$$y = 2x + 2$$





b.) The normal line to the curve $f(x) = 2\sqrt{x}$ when $x = 9$.

First find $f(9)$: $f(9) = 2\sqrt{9} = 2(3) = 6$

So the point we are using is (9, 6)

Then find the derivative. Then find $f'(9)$. The opposite reciprocal will be the slope of the normal line.

$$f(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$$

$$f'(x) = 1x^{-\frac{1}{2}}$$

$$f'(9) = (9)^{-\frac{1}{2}}$$

You can just type this in your calculator... 9 ^ (–1/2) = 1/3. So the slope of the normal line is –3.

Now write the equation using your slope and the given point

$$y - 6 = -3(x - 9)$$

$$y - 6 = -3x + 27$$

$$y = -3x + 33$$