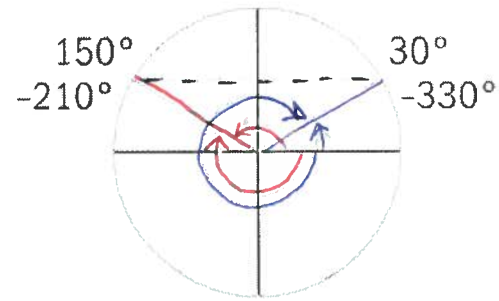
Section 13.2 Solving Equations using the Unit Circle

Suppose you want to solve the equation sin $x = ½$
You must give EVERY place where the sine is ½

There are actually an infinite number of solutions to this equation. You must use your reference angle (the angle whose sine is ½ in Quad I) and what you know about where sine is POSITIVE to name other angles with the same sine.

Suppose your domain is $-360° \le \theta \le 360$

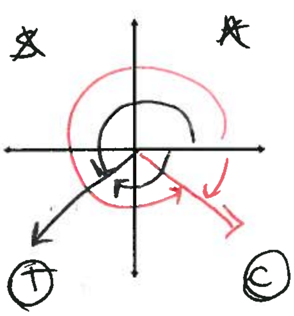


Your reference angle is 30° and sine is positive in Quad I and II.

That means $x = 30°, 150°, -210°, -330°$

Example: Solve the given equations for the given domain

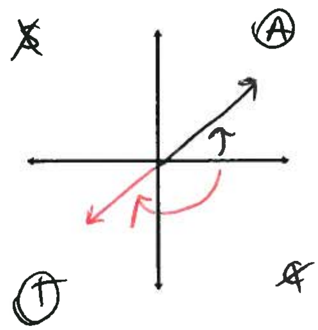
a) $\sin x = -\frac{\sqrt{3}}{2}$, $-2\pi \le \theta \le 2\pi$



Reference Angle: $\pi/3$

$$\boxed{\frac{4\pi}{3},\ -\frac{2\pi}{3},\ \frac{5\pi}{3},\ -\frac{\pi}{3}}$$

b) $3\tan x - \sqrt{3} = 0$, $-\pi \le \theta \le \pi$

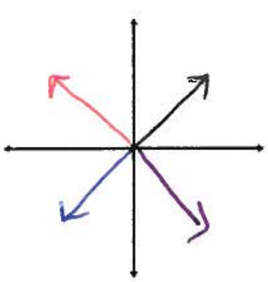


$\tan x = \frac{\sqrt{3}}{3}$

Reference: $\pi/6$

$$\boxed{\pi/6,\ -\frac{5\pi}{6}}$$

c) $\cos^2 x = ¾$, $-180° \le \theta \le 720°$

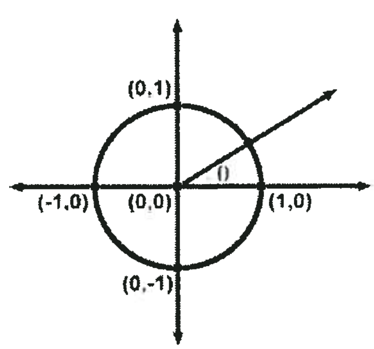


$\cos x = \frac{\pm\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$

All Quads; Ref: 30°

$$\boxed{\begin{array}{l} 30°, 390°, -330°, -690° \\ 150°, 510°, -210°, -570° \\ 210°, 570°, -150°, -510° \\ 330°, 690°, -30°, -390° \end{array}}$$

d) $\sin x = 0$, $-360° \le \theta \le 360°$



points on unit circle
(cos, sin)
y-value = 0

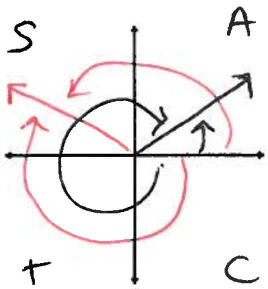
$$\boxed{0°, 180°, 360°, -180°, -360°}$$

Homework C

Example: Solve each equation for the given domain.

a) $\sin 2x = 0.5, \quad -180° \leq x \leq 180$

Pretend this is $\sin \theta = 0.5$



This would be $\theta = 30°, 150°, -210°, -330°$

But $\theta = 2x$

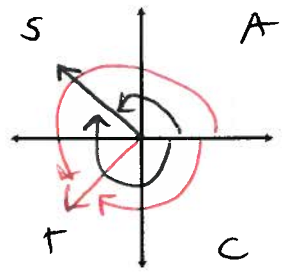
So $2x = 30°, 150°, -210°, -330°$

So $x = 15°, 75°, -105°, -165°$

* must be $-180° \leq x \leq 180$

b) $2\cos(x/2) = -\sqrt{3}, \quad -360° \leq x \leq 360$

Pretend this is $\cos \theta = -\sqrt{3}/2$



This would be $\theta = 150°, 210°, -150°, -210°$

But $\theta = x/2$

So $x/2 = 150°, 210°, -150°, -210°$

So $x = 300°, 420°, -300°, -420°$

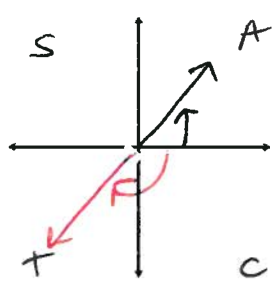
So $x = 300°, -300°$ * must be between $-360° \leq x \leq 360$

c) $\sin \theta = \cos \theta, \quad -\pi \leq \theta \leq \pi$

This is a different kind of equation.
Divide both sides by $\cos \theta$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$



reference angle $= \pi/4$ in Quad I and III

$\theta = \pi/4, -3\pi/4$

d) $2\sin^2 \theta - 3\sin \theta - 2 = 0, \quad 0 \leq \theta \leq 2\pi$

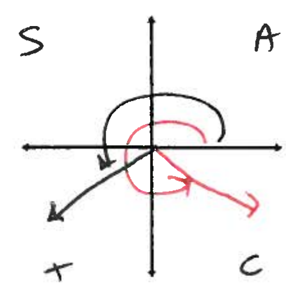
Pretend this is $2x^2 - 3x - 2 = 0$ and factor

$(2x + 1)(x - 2) = 0$

$x = -½$ and $x = 2$

So…
$\sin x = -½$

$\sin x = 2$



To solve $\sin x = -½$ we get a ref of $\pi/6$
$x = 7\pi/6$ and $11\pi/6$

$\sin x = 2$ has no solution

Hmwk D