# 13.2 The Power Rule

**Power Rule:** If $f(x) = ax^n$ then $f'(x) = nax^{n-1}$, where $a$ and $n \in \mathbb{R}$

**Example 1:** Use the power rule to find the derivative of each function below. All final answers should be simplified as much as possible and contain only positive exponents.

a.) $f(x) = x^{12}$

$f'(x) = 12x^{11}$

b.) $f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3x^{-4}$

$f'(x) = \frac{-3}{x^4}$

c.) $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{1}{2\sqrt{x}}$

d.) $f(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$

$f'(x) = -\frac{1}{3}x^{-4/3}$

$f'(x) = \frac{-1}{3x^{4/3}}$ or $\frac{-1}{3\sqrt[3]{x^4}}$

**Definition:** The process of finding the derivative of a function is called **differentiation.**

**More derivative Rules:**

Think about $y = c$, where $c$ is a constant. Think about the slope.

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| **Constant Rule:** If $f(x) = c$, where $c$ is any real number, then $f'(x) = 0$ |

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| **Derivative of a Sum:** If $f(x) = ax^n$ and $g(x) = bx^m$, then $f'(x) + g'(x) = nax^{n-1} + mbx^{m-1}$<br>Essentially, take the derivative of each piece of the function. This works for subtraction too. |

**Example 2:** Differentiate each function.

a.) $f(x) = 4x^3 + 2x^2 - 3$

$f'(x) = 12x^2 + 4x - 0$

$f'(x) = 12x^2 + 4x$

b.) $f(x) = \frac{1}{4}x^3 + \frac{3}{2}x^2 + \frac{1}{3}x$

$f'(x) = \frac{3}{4}x^2 + 3x + \frac{1}{3}$

c.) $f(x) = 3\sqrt[5]{x} + 8$

$f(x) = 3x^{1/5} + 8$

$f'(x) = \frac{3}{5}x^{-4/5} + 0$

$f'(x) = \frac{3}{5x^{4/5}}$ or $\frac{3}{5\sqrt[5]{x^4}}$

c.) $f(x) = (x-2)(x+4)$

$= x^2 + 4x - 2x - 8$

$f(x) = x^2 + 2x - 8$

$f'(x) = 2x + 2$

d.) $f(x) = \frac{4x^3+2x^2-3}{x}$

$f(x) = 4x^2 + 2x - \frac{3}{x}$

$f(x) = 4x^2 + 2x - 3x^{-1}$

$f'(x) = 8x + 2 + 3x^{-2}$

$f'(x) = 8x + 2 + \frac{3}{x^2}$

**Remember:** A derivative can be symbolized as $f'(x)$ or $y'$ or $\frac{dy}{dx}$.

**Definition:** A **second derivative** is the derivative of the derivative function.

It can be symbolized as $f''(x)$ or $y''$ or $\frac{d^2y}{d^2x}$.

FYI... Higher derivatives can be found as well. But in this course we will only do 1st & 2nd derivatives.

**Example 3:** Let $f(x) = -3x^{-4} + 2x$. Then find the following.

a) $f'(x)$

$f'(x) = 12x^{-5} + 2$

$f'(x) = \frac{12}{x^5} + 2$

b) $f'(-2)$

$f'(-2) = \frac{12}{(-2)^5} + 2$

$= \frac{12}{-32} + 2$

$f'(-2) = 1.625$

c) $f''(x)$ = derivative of $f'(x)$

$f''(x) = -60x^{-6} + 0$

$f''(x) = \frac{-60}{x^6}$

d) $f''(-2)$

$f''(-2) = \frac{-60}{(-2)^6}$

$= \frac{-60}{64}$

$f''(-2) = -.9375$

**Example 4:** The height in feet of an arrow after $t$ seconds can be modelled using the formula $s(t) = -16t^2 - 64t + 512$

a) Find the initial height of the arrow. Then find the height after 2 seconds.

Initial height = $s(0)$

$-16(0)^2 - 64(0) + 512 = 512$ feet

Height after 2 seconds = $s(2)$

$-16(2)^2 - 64(2) + 512 = 320$ feet

b) How long does it take for the arrow to hit the ground?

Let $s(t) = 0$

$$-16t^2 - 64t + 512 = 0$$
$$-16(t^2 + 4t - 32) = 0$$
$$-16(t + 8)(t - 4) = 0$$

$t = -8$ or $t = 4$

So it takes 4 seconds to hit the ground.

c) Find $s'(t)$. This represents the velocity function for the arrow.
Then find the speed the arrow is travelling when it hits the ground.

Velocity Function:

$s(t) = -16t^2 - 64t + 512$

$s'(t) = -32t - 64$

Speed when it hits the ground:

From part (b) this happens when $t = 4$

$s'(4) = -32(4) - 64 = -192$ feet per second

d) Find $s''(t)$. This represents the acceleration function for the arrow.

$$s(t) = -16t^2 - 64t + 512$$
$$s'(t) = -32t - 64$$
$$s''(t) = -32 \text{ feet per second}^2$$