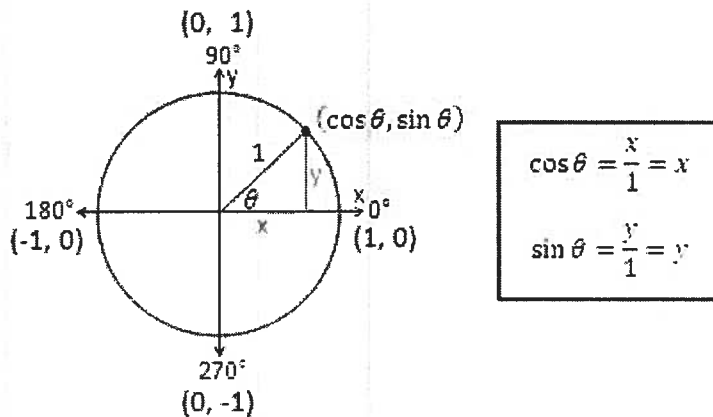


Section 13.1 Using the Unit Circle

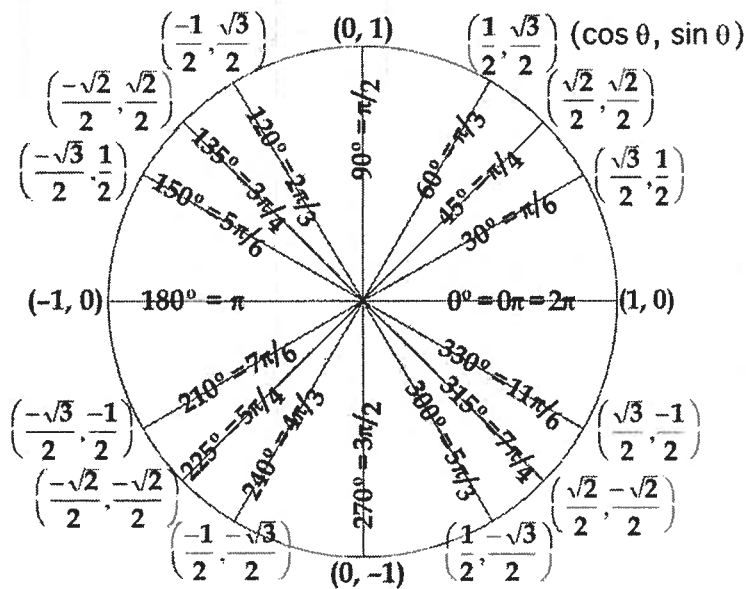
The unit circle has its center at the origin with a radius of 1. The terminal side of any angle θ in standard position will meet the unit circle at a point with coordinates $(\cos \theta, \sin \theta)$

Angles going counterclockwise are positive.
Angles going clockwise are negative.
They can be measured in degrees or radians.

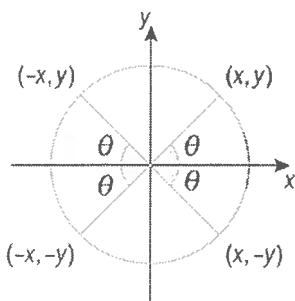
Using symmetry, reference angles (see below), and exact values you can find the exact values of many angles.



Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0



When asked to find trig functions without a calculator you must rely on a table of exact values or your special right triangles and your reference angle. Based on where your terminal side is, find your reference angle (θ') by performing one of the operations below. Reference angles are always positive and are always drawn to the x -axis. Unless they fall on an axis, they will always be less than 90° or $\pi/2$ and normally will be either $\pi/6$, $\pi/4$, or $\pi/3$ (30° , 45° , or 60°). Before you begin to find your reference angle, you must start with a positive angle less than 360° or 2π (either add/subtract 360° or 2π if needed)

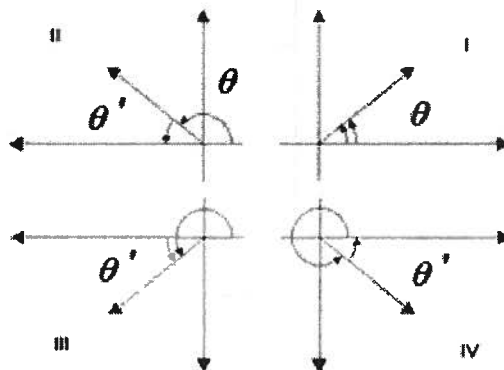


$$\theta' = 180^\circ - \theta$$

$$\theta' = \pi - \theta$$

$$\theta' = \theta - 180^\circ$$

$$\theta' = \theta - \pi$$



$$\theta' = \theta$$

$$\theta' = 360^\circ - \theta$$

$$\theta' = 2\pi - \theta$$

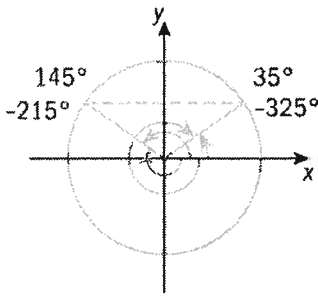
Think "All Students Take Calculus"

That will help you remember the sign of the trig functions in different quadrants. Determine the quadrant of your original angle and then use this chart to determine if its value is positive or negative. Or if you know the sign of your trig function you can get an idea of where it's located.

$\sin \theta$	+	$\sin \theta$	+
$\cos \theta$	-	$\cos \theta$	+
$\tan \theta$	-	$\tan \theta$	+
$\sin \theta$	-	$\sin \theta$	-
$\cos \theta$	-	$\cos \theta$	+
$\tan \theta$	+	$\tan \theta$	-

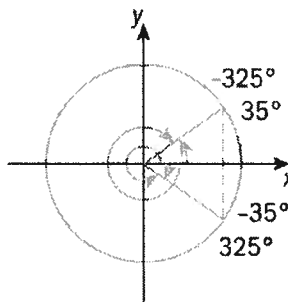
Example: Find three angles that have the same

a) sine 35°



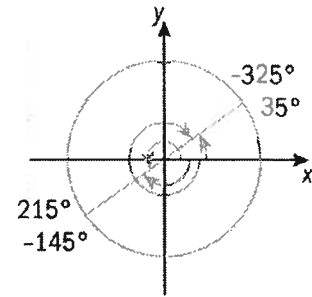
Angles with the same sine have the same y -value on the unit circle. Drawing a horizontal line can help.

b) cosine 35°



Angles with the same cosine have the same x -value on the unit circle. Drawing a vertical line can help.

c) tangent 35°



Angles with the same tangent are in opposite quadrants. Drawing a line through the origin can help.

Related to this example, keep in mind the following useful properties (not given to you on the exam)

$$\sin \theta = \sin (180 - \theta)$$

$$\cos \theta = \cos (-\theta)$$

$$\tan \theta = \tan (180 + \theta)$$

Homework A

Example: Given that $\sin 35^\circ = 0.574$ find each value...

a) $\cos 35^\circ = 0.819$

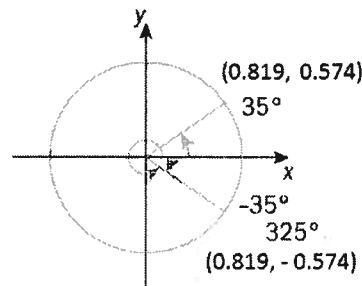
Use the Pythagorean Identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

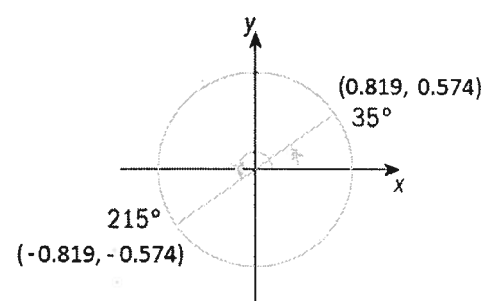
$$(0.574)^2 + \cos^2 35^\circ = 1$$

$$\cos 35^\circ = 0.819$$

b) $\sin 325^\circ = -0.574$



c) $\cos 215^\circ = -0.819$



Example: Given that $\sin A = 0.8$ and $\cos A = 0.6$, find $\cos (180^\circ - A)$

$$\cos (180^\circ - A) = -0.6$$

Example: Given that $\sin \theta = a$ and $\cos \theta = b$, find $\cos (\pi - \theta)$

$$\cos (\pi - \theta) = -b$$

Homework B