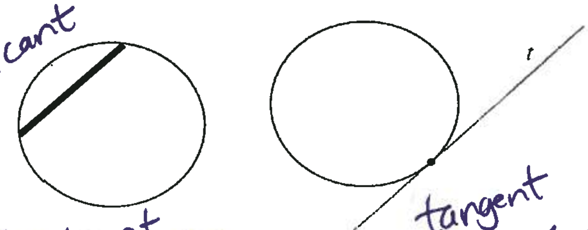
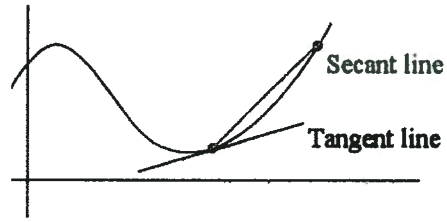
13.1 *The Definition of a Derivative (Day 1)*

Definition: Recall from Geometry your knowledge about **secant lines** and **tangent lines.**
Now let's use it for Calculus

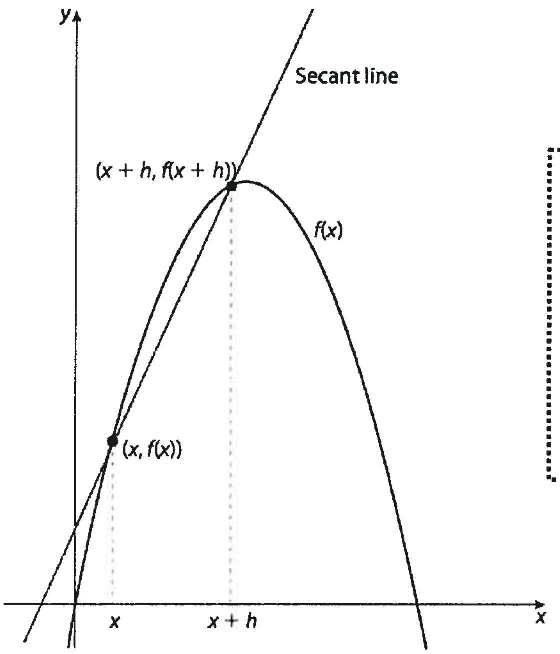


secant
touches at 2 pts on the circle

tangent
(touches at 1 pt)



Definition: The average rate of change of a function is found by finding the slope of a secant line.



Definition: The gradient of a secant line is the same as the slope between two points on the line.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

(Also known as the difference quotient from Pre-Calculus)

We will deal with this more later...

Example 1: Find the slope of the secant line for the function $f(x) = x^2$ between (1,1) and (1.2, 1.44)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1.44 - 1}{1.2 - 1} = \frac{0.44}{0.2} = 2.2$$

Example 2: **Fill in the table below for $f(x) = x^2$ where A = (1,1) giving your answers to the nearest ten-thousandths place.**

x	$f(x) = x^2$	$B(x, f(x))$	Slope of secant line between A and B
1.5	$(1.5)^2 = 2.25$	(1.5, 2.25)	$\frac{2.25-1}{1.5-1} = \frac{1.25}{0.5} = 2.5$
1.1	$(1.1)^2 = 1.21$	(1.1, 1.21)	$\frac{1.21-1}{1.1-1} = \frac{0.21}{0.1} = 2.1$
1.01	$(1.01)^2 = 1.0201$	(1.01, 1.0201)	$\frac{1.0201-1}{1.01-1} = \frac{0.0201}{0.01} = 2.01$
1.001	$(1.001)^2 = 1.002001$	(1.001, 1.002001)	$\frac{1.002001-1}{1.001-1} = \frac{0.002001}{0.001} = 2.001$
1.0001	$(1.0001)^2 = 1.00020001$	(1.0001, 1.00020001)	$\frac{1.00020001-1}{1.0001-1} = 2.0001$

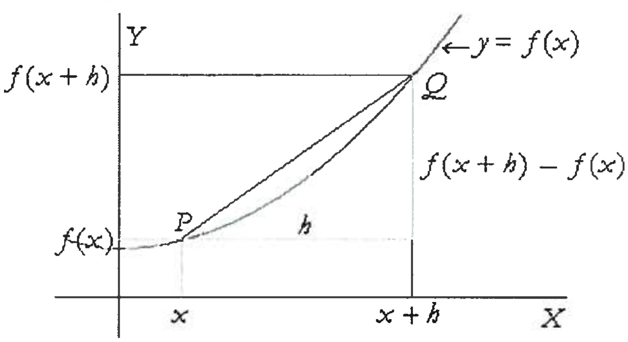
Notice that as the distance between the x-values decreases the value of the slope seems to be getting closer and closer to a value.

That value is the slope of the tangent. The derivative represents a gradient function that you can use the find the slope of the tangent at any given x-value.

In this case

slope = 2

13.1 The Definition of a Derivative ~~(Day 2)~~ cont,



If the distance between the x-values of two points P and Q (represented by h) becomes closer and closer to zero then it becomes a tangent line.

That slope is represented by

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\lim_{h \to 0}$ is called a limit.

It involves the process of making h closer and closer to a number, in this case zero.

The function that comes about when doing the slope using that method is called a derivative. If that function is evaluated at a given x-value then that is the slope of the tangent line at that x-value.

A derivative can be symbolized by $f'(x)$ or y' or $\frac{dy}{dx}$.

a b

Example: **Use the definition of derivative to find the derivative of $f(x) = 3x^2 + x$ and hence find the gradient of the tangent line when $x = 4$ (find the value of $f'(4)$)**

Definition of Derivative:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{[3(x+h)^2 + (x+h)] - [3x^2 + x]}{h}$$

$$\lim_{h \to 0} \frac{[3(x^2 + 2xh + h^2) + (x+h)] - [3x^2 + x]}{h}$$

$$\lim_{h \to 0} \frac{[3x^2 + 6xh + 3h^2 + x + h] - [3x^2 + x]}{h}$$

$$\lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 3x^2 - x}{h}$$

$$\lim_{h \to 0} \frac{6xh + 3h^2 + h}{h}$$

$$\lim_{h \to 0} 6x + 3h + 1$$

Now let $h = 0$, so $6x + 3(0) + 1 = 6x + 1$

a So the derivative function is $6x + 1$

The value (the gradient) at $x = 4$ would be $6(4) + 1 = 25$

b So $f'(4) = 25$