12.5 Application of Vectors

Objective:

- **Apply vectors to real-life situations including quantities such as displacements and velocities.**

Basics: Velocity is displacement over time. When writing position vectors keep in mind the horizontal movement (East = positive/West = negative) is the horizontal component and the vertical movement (North = positive/South = negative) is the vertical component. When written as a vector equation of a line $\mathbf{r} = \mathbf{a} + t\,\mathbf{b}$, the directional vector $\mathbf{b}$ is the velocity vector.

Example: The position vector of a ship is 25 km east and 50 km north and the position vector of a lighthouse is 30 km south and 55 miles east.

a) What is the position of the ship relative to the lighthouse?
b) What is the exact distance between the two?

a) Ship vector = $\begin{pmatrix} 25 \\ 50 \end{pmatrix}$ Lighthouse vector = $\begin{pmatrix} 55 \\ -30 \end{pmatrix}$

b) To find the distance, compute the magnitude of the relative position vector.

Relative Position: $\begin{pmatrix} 25 \\ 50 \end{pmatrix} - \begin{pmatrix} 55 \\ -30 \end{pmatrix} = \begin{pmatrix} -30 \\ 80 \end{pmatrix}$

$\sqrt{(-30)^2 + (80)^2} = \sqrt{7300} \approx 85.4$ ~~miles~~ km

30 ~~miles~~ km West; ~~75~~ 80 ~~miles~~ km North

Example: Using the information in the last example…

a) If the boat arrives at the lighthouse 10 hours later, what is its velocity?
b) What is the equation representing the ship's path from the lighthouse at the same velocity.

a) Velocity = displacement/time

$\frac{1}{10}\begin{pmatrix} -30 \\ 80 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$ km / h or km h^{-1}

$\sqrt{(-3)^2+(8)^2} = \sqrt{73} = 8.5$

b) $\mathbf{r} = \mathbf{a} + t\,\mathbf{b}$

$\mathbf{r} = \begin{pmatrix} 55 \\ -30 \end{pmatrix} + t\begin{pmatrix} -3 \\ 8 \end{pmatrix}$

Example: Two particles have positions defined as $\mathbf{r}_1 = \begin{pmatrix} -8 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 2 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ where $t \geq 0$ is measured in minutes and the distance is measured in meters.

a) How far apart at the two particles initially (time = 0)?

This is like the first example above, except we are given the vectors in the equations…

Relative Position: $\begin{pmatrix} -8 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -7 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \end{pmatrix}$ $\sqrt{(-10)^2 + (10)^2} = \sqrt{200} \approx 14.1$ meters

b) How far apart are the particles after 3 ~~seconds~~ minutes?

Plug in $t = 3$ into each equation and then find the relative position and magnitude

$\mathbf{r}_1 = \begin{pmatrix}-8\\3\end{pmatrix} + (3)\begin{pmatrix}4\\3\end{pmatrix}$ $\quad$ $\mathbf{r}_2 = \begin{pmatrix}2\\-7\end{pmatrix} + (3)\begin{pmatrix}2\\5\end{pmatrix}$

$\mathbf{r}_1 = \begin{pmatrix}-8\\3\end{pmatrix} + \begin{pmatrix}12\\9\end{pmatrix}$ $\quad$ $\mathbf{r}_2 = \begin{pmatrix}2\\-7\end{pmatrix} + \begin{pmatrix}6\\15\end{pmatrix}$

$\mathbf{r}_1 = \begin{pmatrix}4\\12\end{pmatrix}$ $\quad$ $\mathbf{r}_2 = \begin{pmatrix}8\\8\end{pmatrix}$

$$\begin{pmatrix}4\\12\end{pmatrix} - \begin{pmatrix}8\\8\end{pmatrix} = \begin{pmatrix}-4\\4\end{pmatrix}$$

$\sqrt{(-4)^2 + (4)^2} = \sqrt{32} \approx 5.7$ meters

c) If the particles continue on their initial path, will they collide? If so, when and where?

If they collide, then their position after t minutes will be the same. So set them equal.

$\begin{pmatrix}-8\\3\end{pmatrix} + t\begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}2\\-7\end{pmatrix} + t\begin{pmatrix}2\\5\end{pmatrix}$ which would be $\begin{array}{lll} -8 + 4t = 2 + 2t & \text{where} & t = 5 \\ 3 + 3t = -7 + 5t & \text{where} & t = 5 \end{array}$

Since both equations solve to give you the same value, they collide.

***When? They collide after 5 minutes.

***Where? Do what was done in part b above. You really only need to plug it into one of the equations.

Plug in $t = 5$ into either equation to find its position at collision.

$\mathbf{r}_1 = \begin{pmatrix}-8\\3\end{pmatrix} + (5)\begin{pmatrix}4\\3\end{pmatrix}$ $\quad$ $\mathbf{r}_2 = \begin{pmatrix}2\\-7\end{pmatrix} + (5)\begin{pmatrix}2\\5\end{pmatrix}$

$\mathbf{r}_1 = \begin{pmatrix}-8\\3\end{pmatrix} + \begin{pmatrix}20\\15\end{pmatrix}$ $\quad$ $\mathbf{r}_2 = \begin{pmatrix}2\\-7\end{pmatrix} + \begin{pmatrix}10\\25\end{pmatrix}$ $\quad$ They collide at $\begin{pmatrix}12\\18\end{pmatrix}$

$\mathbf{r}_1 = \begin{pmatrix}12\\18\end{pmatrix}$ $\quad$ $\mathbf{r}_2 = \begin{pmatrix}12\\18\end{pmatrix}$

d) How fast are the particles travelling?

The magnitudes of the direction vectors are the magnitudes of the velocity vectors.

Particle 1:

$$\begin{pmatrix}4\\3\end{pmatrix} = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5 \text{ meters/min}$$

Particle 2:

$$\begin{pmatrix}2\\5\end{pmatrix} = \sqrt{(2)^2 + (5)^2} = \sqrt{29} = 5.4 \text{ meters/min}$$