

Section 12.4 χ^2 Test of Independence

The table to the right shows the results of a sample of 400 randomly selected adults classified according to gender and regular exercise.

	Regular exercise	No regular exercise
Male	110	106
Female	98	86

We call this a 2 x 2 contingency table.

We may be interested in how the variables gender and regular exercise are related. The variables may be dependent, for example, females may be more likely to exercise regularly than males. Alternatively, the variables may be independent, which means the gender of a person has no effect on whether they exercise regularly.

The chi-squared or χ^2 test is used to determine whether 2 variables from the same sample are independent or not. The χ^2 test examines the difference between the observed values we obtained from our sample, and the expected values we have calculated.

Today we will let the calculator do all the work for us. In the next section we will do some of the work by hand. On the exam, most of the work will be done on the graphing calculator.

Formal test for independence

A small value of χ^2 indicates that two variables are independent, while a large value of χ^2 indicates that the variables are not independent. We will now consider a more formal test which determines how large χ^2 must be for us to conclude that the variables are not independent. This is known as the critical value of χ^2 .

The critical value of χ^2 depends on:

1. the size of the contingency table, measured by degrees of freedom $(r - 1)(c - 1)$
2. the significance level used

Significance level (α)

As χ^2 gets larger, it becomes increasingly unlikely that the variables involved are independent. The significance level indicates the minimum acceptable probability that the variables are independent.

We usually use either 10%, 5%, or 1% for the significance level, often given as $\alpha = 0.10$, $\alpha = 0.05$, and $\alpha = 0.01$

For a given significance level and degrees of freedom, the table to the right gives the critical value of χ^2 , above which we conclude the variables are not independent.

For example, at a 5% significance level with $df = 1$, the critical value of 3.84. This means that at a 5% significance level, the departure between the observed and expected values is too great if $\chi^2_{calc} > 3.84$.

Likewise, at a 1% significance level with $df = 7$, the departure between the observed and expected values is too great if $\chi^2_{calc} > 18.48$.

Degrees of freedom (df)	Significance level		
	10%	5%	1%
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21

Luckily, on the IB test, we will be given the critical value to use – we will not have to look at a table to find it.

p-value

When finding χ^2 on your calculator, a p -value is also provided. This can be used, together with the χ^2 value and the critical value, to determine whether or not to accept that the variables are independent.

For a given contingency table, the p -value is the probability of obtaining observed values as far or further from the expected values, assuming the variables are independent. The p -value tells you if your test results are significant or not.

If the p -value is smaller than the significance level, then it is sufficiently unlikely that we would have obtained the observed results if the variables were independent. We therefore conclude that the variables are not independent.

χ^2 test of independence

When we conduct the χ^2 test of independence, we are trying to figure out if two variables occur independently of each other. In our test, we will need to create our hypothesis.

Null (H_0) hypothesis: H_0 states that the data sets are independent

Alternative (H_1) hypothesis: H_1 states that the data sets are not independent

For example, the hypothesis for gender and regular exercise could be:

Null (H_0) hypothesis: Gender and regular exercise are independent

Alternative (H_1) hypothesis: Gender and regular exercise are not independent

Note that we do not say that the two variables are dependent. Our test is not concluding whether something is dependent or not – it just telling us if two things are independent or not independent. Think of it like a jury determining if someone is guilty or not guilty. We don't say that someone is innocent – we can just say that they are not guilty.

To figure out whether our null or alternative hypothesis is correct, we will have to compare our χ^2_{calc} to the given critical value and our p -value to the given significance level.

To perform the formal χ^2 test of independence, use the following steps:

1. Write the null (H_0) and alternative (H_1) hypotheses.
2. Calculate χ^2_{calc} using your GDC. (Go to Stat, Tests, χ^2 - Test)
3. Determine
 - a. the p -value using your GDC
 - b. the critical value (given on the IB test)
4. Compare
 - a. the p -value against the significance level (given on the IB test)
 - i. If the p -value is less than the significance level, then reject the null hypothesis.
 - ii. If the p -value is more than the significance level, then do not reject the null hypothesis.
Think if p is low, reject. (p is a probability so that makes sense)
 - b. χ^2_{calc} against the critical value (given on the IB test)
 - i. If χ^2_{calc} is less than the critical value, then do not reject the null hypothesis.
 - ii. If χ^2_{calc} is more than the critical value, then reject the null hypothesis.
Think if chi is high, reject. (it rhymes and is similar to the p statement above so it should be easy to remember... just be careful because the high/low is opposite)

Example 1. A real estate developer conducted a survey. He asked 100 people who live in the city and 100 people who live in the suburbs if they prefer bicycling, driving walking or taking public transportation to work. The results are shown in the table below.

	City	Suburbs
Bicycle	15	20
Drive	35	30
Walk	15	20
Public Transportation	35	30

A χ^2 test was conducted at the 1% , 5%, and 10% significance level.

- Write down the null and alternate hypothesis for this test.
- Use your calculator to find the p-value for this test.
- Determine whether the null hypothesis is justified for $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$. Give a reason for your answer.

- State the null hypothesis and the alternative hypothesis

H_0 : Whether people live in the city or suburbs is independent of their choice of getting to work

H_1 : Whether people live in the city or suburbs is not independent of their choice of getting to work

- Write down the p -value for this data. Remember to use 3 sig figs.

Perform the steps on the Calculator Instruction Sheet. Enter the table in Matrix A. Then go to Stats to perform the Test.

p -value: 0.532

- Using the $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$ significance levels, comment on your result.

$\alpha = 0.01$: Since the p -value of 0.532 > 0.01, we fail to reject the null hypothesis. There is sufficient evidence to conclude that whether people live in the city or suburbs is independent of their choice of getting to work.

$\alpha = 0.05$: Since the p -value of 0.532 > 0.05, we fail to reject the null hypothesis. There is sufficient evidence to conclude that whether people live in the city or suburbs is independent of their choice of getting to work.

$\alpha = 0.10$: Since the p -value of 0.532 > 0.10, we fail to reject the null hypothesis. There is sufficient evidence to conclude that whether people live in the city or suburbs is independent of their choice of getting to work.

Example 2. Members of a club are required to register for one of three games: billiards (pool), snooker (type of pool game), or darts. The number of club members of each gender choosing each game in a particular year is shown in the table. The results are shown in the table below

	Billiards	Snooker	Darts	Total
Male	39	16	8	63
Female	21	14	17	52
Total	60	30	25	115

A χ^2 test was conducted at the 1% , 5%, and 10% significance level.

- a. Write down the null and alternate hypothesis for this test.
 - b. Use your calculator to find the p-value for this test.
 - c. Determine whether the null hypothesis is justified for $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$. Give a reason for your answer.
- a. State the null hypothesis and the alternative hypothesis

H_0 : Preference of chosen game and gender are independent.

H_1 : Preference of chosen game and gender are not independent.

- b. Write down the p -value for this data. Remember to use 3 sig figs.

p -value: 0.0203

- a. Using the $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$ significance levels, comment on your result.

$\alpha = 0.01$: Since the p -value of 0.0203 $>$ 0.01, we fail to reject the null hypothesis. There is sufficient evidence to conclude that the preference of chosen game and gender is independent.

$\alpha = 0.05$: Since the p -value of 0.0203 \nless 0.05, we reject the null hypothesis. There is sufficient evidence to conclude that the preference of chosen game and gender are not independent.

$\alpha = 0.10$: Since the p -value of 0.0203 \nless 0.10, we reject the null hypothesis. There is sufficient evidence to conclude that the preference of chosen game and gender are not independent.

How to perform a Chi-Squared Test of Independence on a TI-84 Plus Calculator

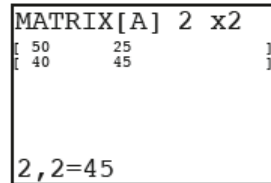
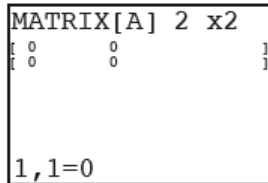
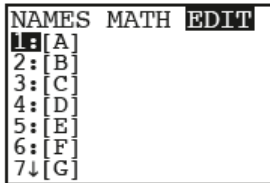
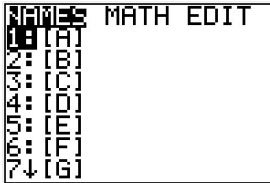
First... enter your contingency table as Matrix A. If your table has row or column totals do not include those.

Go to Matrix (Hit 2nd and x⁻¹)

Go to Edit and choose Matrix [A]

Change the size if need be by changing the numbers at the top. The first number is the number of rows. The second number is the number of columns.

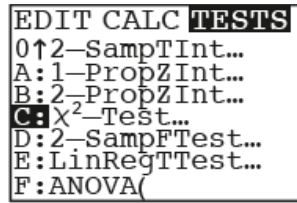
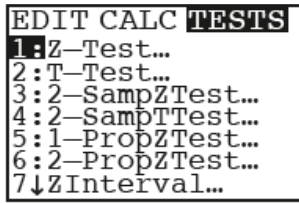
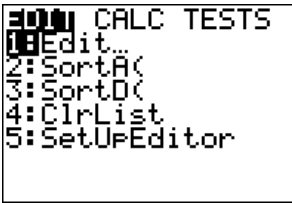
Enter the numbers from the contingency table. You can use the Enter key or the arrows to move from cell to cell.



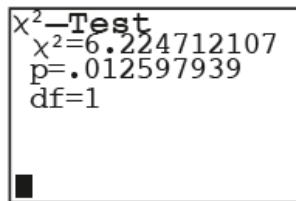
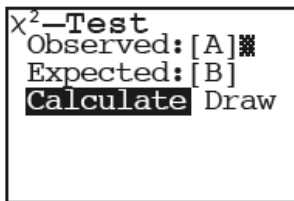
Quit (Hit 2nd and MODE)

Go to Stats

Go to Tests and choose χ^2 - Test.



Make sure your screen looks like the first one below. Hit enter on Calculate and you should get a screen like the second one below.



The χ^2 value gets compared with the given critical value. The p -value gets compared with the given significance level (as a decimal).

Compare

- a. the p -value against the significance level (given on the IB test)
 - i. If the p -value is less than the significance level, then reject the null hypothesis.
 - ii. If the p -value is more than the significance level, then do not reject the null hypothesis.

Think if p is low, reject. (p is a probability so that makes sense)
- b. χ^2_{calc} against the critical value (given on the IB test)
 - i. If χ^2_{calc} is less than the critical value, then do not reject the null hypothesis.
 - ii. If χ^2_{calc} is more than the critical value, then reject the null hypothesis.

Think if chi is high, reject. (it rhymes and is similar to the p statement above so it should be easy to remember... just be careful because the high/low is opposite)