12.2 Addition and Subtraction of Vectors

Objectives:

- **Add and subtract vectors in rectangular form**
- **Use the zero vector to determine equilibrium**
- **Use vectors to deduce geometric results**

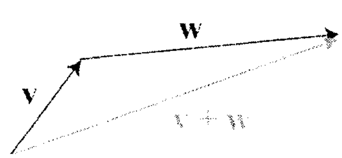
Definitions: Vector addition corresponds to combined or net effects.
For example, if **v** and **w** are force vectors, then the resultant vector **v** + **w** represents net force.
Vector subtraction would be defined as **v** – **w** or as adding a negative vector **v** + (– **w**).

Arithmetically: Given the vectors **v** = 3**i** + 4**j** and **w** = –2**i** + 5**j** , find the following…

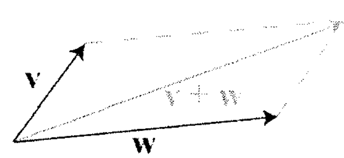
v + **w**	**v** – **w**	2 **v** + **w**
(3**i** + 4**j**) + (–2**i** + 5**j**)	(3**i** + 4**j**) – (–2**i** + 5**j**)	2(3**i** + 4**j**) + (–2**i** + 5**j**)
i + 9**j**	5**i** – **j**	(6**i** + 8**j**) + (–2**i** + 5**j**)
		4**i** + 13**j**

Geometrically: Vector addition can be thought of as first move along vector **v** followed by move along vector **w**. This is the triangle law. This is good for representing sequential effects and displacements. Because vector addition is commutative (**v** + **w** = **w** + **v**), it also gives rise to the parallelogram law. This is better for representing simultaneous effects and net force.

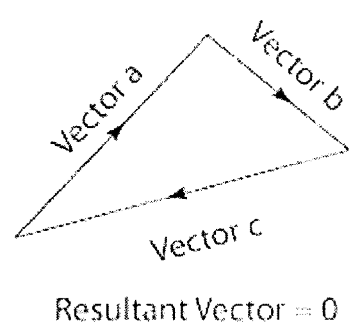
Triangle Law



Parallelogram Law



The Zero Vector: In the figure at the right, **a** + **b** = **c**. Since the overall journey results in returning to the starting position, the resultant vector **c** must equal 0. This would be written as **a** + **b** + **c** = **0**



Equilibrium is the state where a number of forces are in balance. This would be an example of a resultant vector of 0.

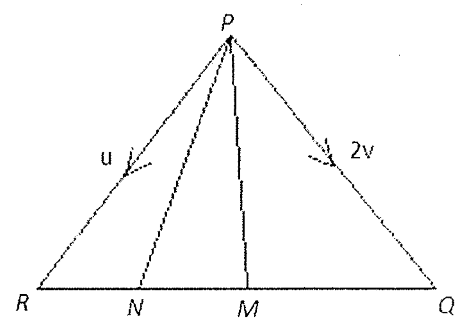
The zero vector is written as such

$$\mathbf{0} = \begin{pmatrix}0\\0\end{pmatrix} \text{ in two dimensions and } \begin{pmatrix}0\\0\\0\end{pmatrix} \text{ in three dimensions}$$

Homework G

Geometrical Proofs: Using vector addition, subtraction, and scalar multiples to deduce some geometrical results.

Example: In the diagram at the right,



$\overrightarrow{PR} = \mathbf{u}$ and $\overrightarrow{PQ} = 2\mathbf{v}$ and

M is the midpoint of *RQ* and *N* is the midpoint *RM*.

Find the following in terms of **u** and **v**.

a) $\overrightarrow{RQ}$ b) $\overrightarrow{RN}$ c) $\overrightarrow{PN}$

Solutions:

a) $\overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$ Triangle Law of Vector Addition

$= \overrightarrow{-PR} + \overrightarrow{PQ}$ Substitute a negative vector

$= -\mathbf{u} + 2\mathbf{v}$ Substitute

$= 2\mathbf{v} - \mathbf{u}$

b) $\overrightarrow{RN} = \frac{1}{4}\overrightarrow{RQ}$

$= \frac{1}{4}(2\mathbf{v} - \mathbf{u})$

$= \frac{1}{2}\mathbf{v} - \frac{1}{4}\mathbf{u}$

c) $\overrightarrow{PN} = \overrightarrow{PR} + \overrightarrow{RN}$

$= \mathbf{u} + \left(\frac{1}{2}\mathbf{v} - \frac{1}{4}\mathbf{u}\right)$

$= \frac{3}{4}\mathbf{u} + \frac{1}{2}\mathbf{v}$

Homework H