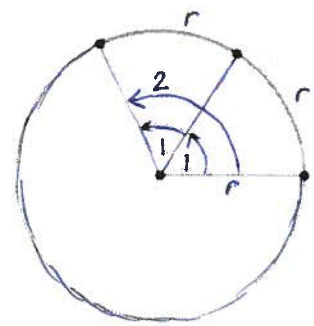
Section 11.7 Radians, Arcs, and Sectors

* change calc to radian mode

One way an angle can be measured is in **degrees**. 1 Degree is 1/360 of a circle The Babylonians believed that there were 360 days in a year and hence used 360° to represent one revolution. But this number is a somewhat arbitrary measure.

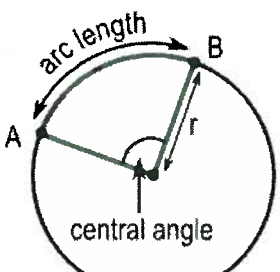
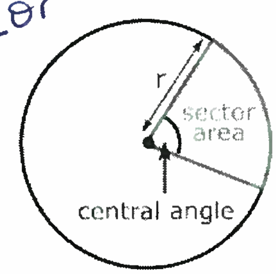


Radians are another unit of measurement that can simplify many formulas used in calculus and physics. Radian measures are directly related to measurements within a circle.

One radian is defined as the size of the central angle subtended by an arc which is the same length as the radius of the circle. Two radians is the size of the central angle subtended by an arc with a length twice the radius of the circle.

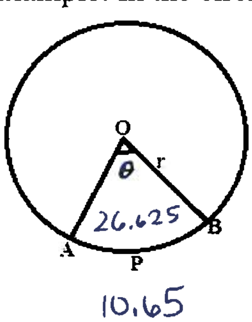
One complete turn around the circle is subtended by an arc equal in length to the circumference of the circle, $2\pi r$. Therefore, the angle which subtends the circumference of the circle is 2π radians.

Any central angle in a circle is a fraction of 2π, so you can calculate the length of a subtended arc as the fraction of the circumference. Similarly the area of a sector will be a fraction of the area of the circle.

Arc Length:	Area of a Sector:
$\left(\frac{\theta}{2\pi}\right)(2\pi r) = r\theta$  where r is the radius and θ is the central angle measure in radians	or $\frac{1}{2}\theta r^2$ $\left(\frac{\theta}{2\pi}\right)(\pi r^2) = \frac{\theta r^2}{2}$  where r is the radius and θ is the central angle measure in radians

Note: an angle which is a multiple of π is assumed to be measured in radians. Otherwise the angle will be written as, for example, 1.3 radians or 1.3 rad. Exact values will be given in terms of π.

Example: In the circle below, arc AB = 10.65 mm and the area of sector AOB = 26.625 mm^2.



a) Find the central angle θ and the radius r.

$A = \frac{\theta r^2}{2}$ arc length $= r\theta$

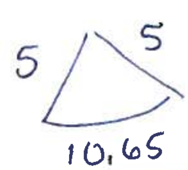
$26.625 = \frac{\theta r^2}{2}$ $10.65 = r\theta$

$53.25 = (\theta r) r$

$53.25 = (10.65) r$

$\boxed{r = 5 \text{ mm}}$ $\boxed{\theta = 2.13 \text{ radians}}$

b) Find the perimeter of sector AOB



$\boxed{20.65 \text{ mm}}$

Exercise 11 K

A full revolution gives a central angle of 2π radians as well as 360° Using that fact we can develop a ratio to do conversions:

$$1 \text{ Radian} = \frac{360°}{2\pi} \text{ or } \frac{180°}{\pi}$$

Example: Perform the following conversions.

a) Convert $\frac{2\pi}{5}$ to degrees

$$\frac{2\pi}{5} \cdot \frac{180°}{\pi} = \frac{2\cancel{\pi}}{\cancel{5}} \cdot \frac{\cancel{180}^{36}°}{\cancel{\pi}} = 72°$$

b) Convert –12° to radians

$$-12° \cdot \frac{\pi}{180°} = \frac{-12°\pi}{180°} = \frac{-\cancel{12}\pi}{\cancel{180}_{15}} = \boxed{\frac{-\pi}{15}}$$

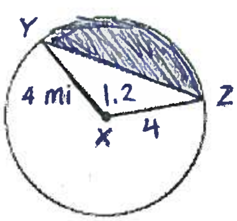
Exercise 11 L

In Section 11.1 you learned some "special" angles in right-angled triangles. These angles and their multiples are often used. It would be helpful if you memorize their radian equivalents so you don't have to convert them every time. These angles are usually written in exact radian form using π.

Be careful to note whether your angles are given or should be found in degrees or radians. Also be sure your calculator is in the appropriate mode, Degree or Radian.

degrees	radians
0	0
30	$\pi/6$
45	$\pi/4$
60	$\pi/3$
90	$\pi/2$
120	$2\pi/3$
135	$3\pi/4$
180	π
225	$5\pi/4$
270	$3\pi/2$
315	$7\pi/4$
360	2π

Example: Find the area of the shaded region to 3 significant figures if $\theta = 1.2$ radians.



Area of Sector – Area of Triangle

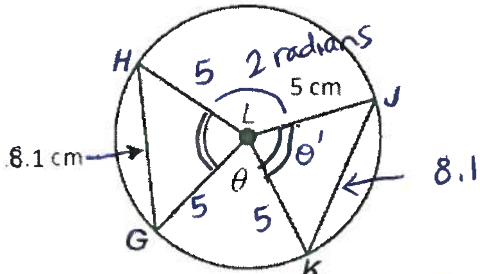
$\frac{1}{2}(1.2)(4)^2 \quad - \quad \frac{1}{2}(4)(4)\sin 1.2$ ← radians!

(change calc mode or convert to degrees)

$9.6 \quad - \quad 7.456\ldots$

$\boxed{2.14 \text{ mi}^2}$

Example: Find the area of ΔJKL. Then find θ and length of arc GK. Give all answers to 3 significant figures.



Use Cosine Rule to find θ'

radian mode!

$8.1^2 = 5^2 + 5^2 - 2(5)(5)\cos\theta'$

$-.3122 = \cos\theta'$

$\theta' = 1.89$

length GK

θr

$(.503)(5)$

$\boxed{2.52 \text{ cm}}$

Area of ΔJKL

$\frac{1}{2}(5)(5)\sin 1.89$

$\boxed{11.9 \text{ cm}^2}$

$\theta = 2\pi - 1.89 - 1.89 - 2$

$\boxed{\theta = .503 \text{ radians}}$

Exercise 11 M