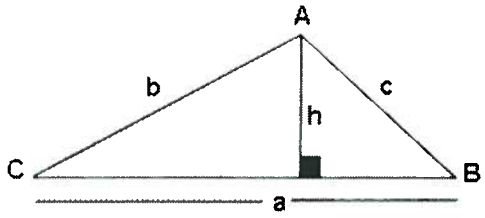
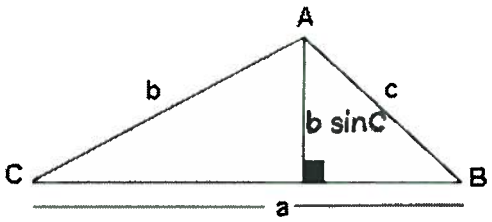
Section 11.6 Area of a Triangle

The traditional formula for the area of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height perpendicular to the base)

Look at triangle ABC.
Its area would be defined as $A = \frac{1}{2}ah$



Substitute that expression for h.



According to the triangle above $\sin C = \frac{h}{b}$.
Similarly, $h = b \sin C$.

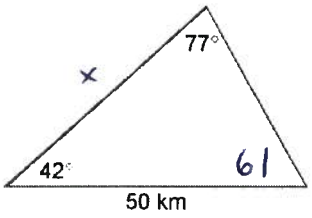
Now it can be said that $\text{Area} = \frac{1}{2}a(b \sin C)$

← SAS!

Essentially if you know the lengths of two sides of a triangle and the measure of the angle between the two sides, you can find the area of the triangle. That would give me three formulas…

$$\text{Area} = \frac{1}{2}ab \sin C \qquad \text{Area} = \frac{1}{2}ac \sin B \qquad \text{Area} = \frac{1}{2}bc \sin A$$

Example: Find the area of the triangle below.



I don't have SAS
So find another side

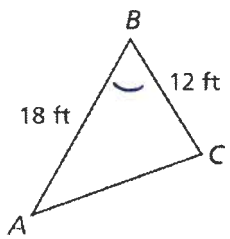
$\frac{\sin 61}{x} = \frac{\sin 77}{50}$

$x = 44.9$

$A = \frac{1}{2}(44.9)(50)\sin 42$

$A = 751.1 \text{ km}^2$

Example: The triangle below has an area of 94 ft^2.
Find $\hat{B}$.



$A = \frac{1}{2}ac \sin B$

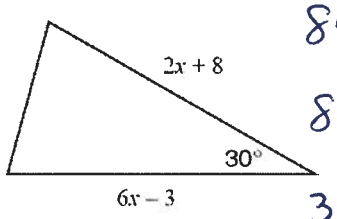
$94 = \frac{1}{2}(12)(18)\sin B$

$94 = 108 \sin B$

$\frac{47}{54} = \sin B$

$B = 60.5°$ or $119.5°$

Example: The triangle below has an area of 84 ft^2.
Find the value of x.



$84 = \frac{1}{2}(2x+8)(6x-3)\sin 30$

$84 = \frac{1}{4}(12x^2+42x-24)$

$336 = 12x^2+42x-24$

$12x^2+42x-360 = 0$

Quad Form ←

$x = -7.5$ or 4

Exercise 11J

a cd, 2 – 6