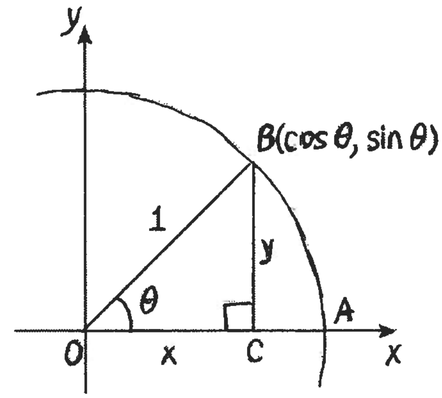
Section 11.3 Using the Coordinate Axes in Trigonometry

If a circle has center of (0, 0) and a radius of 1 it is called a unit circle.

According to the picture, your opposite side would be y and your adjacent side would be x. The hypotenuse would also be the radius of the unit circle which is 1.



That means for this picture…

$$\sin\theta = \frac{y}{1} \qquad \cos\theta = \frac{x}{1} \qquad \tan\theta = \frac{y}{x}$$

or

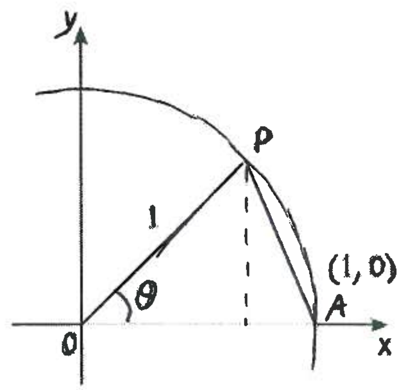
$$\sin\theta = y \qquad \cos\theta = x \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

That also makes your point $P(x, y)$ be (cos θ, sin θ). That's ABC order just like (x, y)!

Example: Use the diagram at the right to

a) Find the coordinates of point P to 3 significant figures if $\theta = 33°$.

$P(x,y) = (\cos 33, \sin 33) = \boxed{(.839, .545)}$



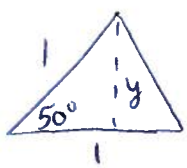
b) Find the value of θ if the coordinates of point P are (0.225, 0.974). (Give your answer to the nearest degree)

$\cos\theta = .225 \quad \theta = 77.0$

$\sin\theta = .974 \quad \theta = 76.9$

$\boxed{\theta = 77°}$

c) Find the area of ΔAOP if $\theta = 50°$. (Give your answers to 3 significant figures)



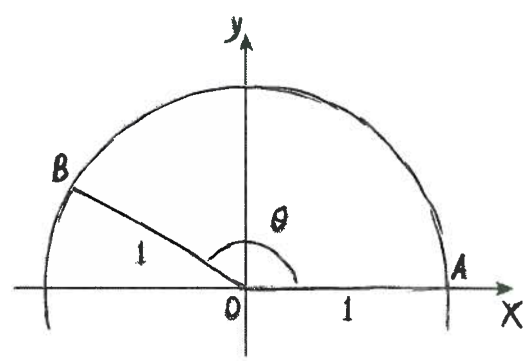
$y = \sin 50° = .766$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(1)(.766) = \boxed{.383}$

Exercise 11 D

When working with obtuse angles (angles between 90° and 180° in quadrant 2), it can be helpful to think of how they relate to acute angles in quadrant 1.

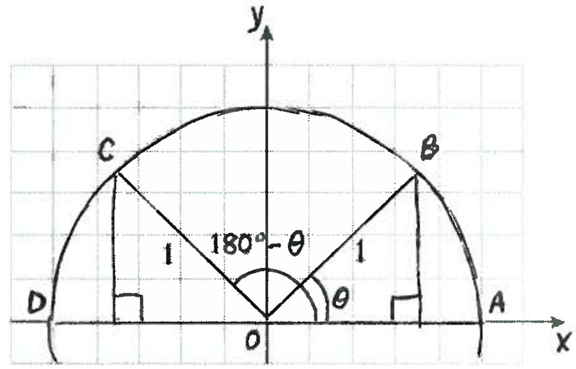


Investigation – Obtuse Angles pg 375

For supplementary angles A and B (angles that add to 180°), sin A = sin B and cos A = –cos B
For any angle θ, sin θ = sin (180° – θ), and cos θ = – cos (180° – θ)

Example: Use the diagram at the right to

a) Find the coordinates of points B and C if $\theta = 25°$
(Give your answer to 3 significant figures)



$B = (\cos 25, \sin 25) = (.906, .423)$

$C = (-.906, .423)$

b) Find the value of θ if C is (–0.819, 0.574)
(Give your answer to the nearest tenth of a degree)

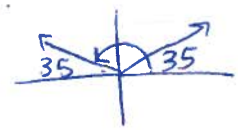
$C = (-.819, .574)$
$B = (.819, .574)$

or $\cos^{-1}(-.819)$
145°

$\cos\theta = .819$
35.0

$\sin\theta = .574$
35.0

Q2!

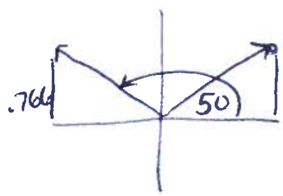


180 − 35
$\boxed{145°}$

Example: Find the sine (to 4 significant figures) of 50° and state the obtuse angle that has the same sine.

$\sin 50° = .7660$

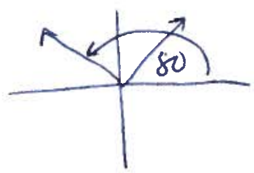
180 − 50
$\boxed{130°}$



Example: Find one acute and one obtuse value for A if sin A = 0.985

$\sin^{-1}(.985) = 80.1 \approx \boxed{80°}$

180 − 80 $\boxed{100°}$



Exercise 11E

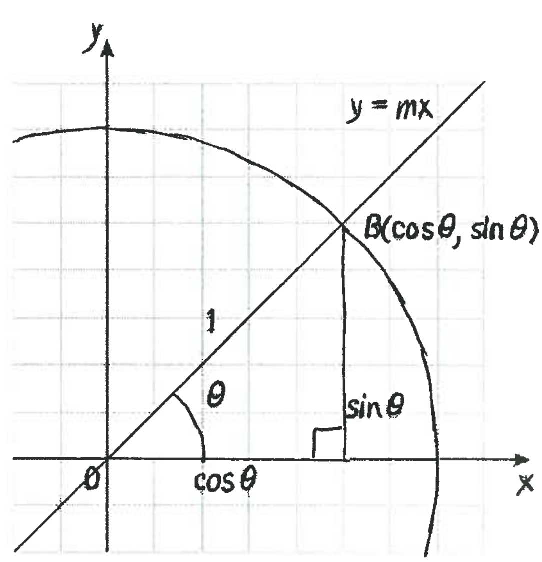
Look at what happens if the line $y = mx$ with gradient (slope) of m intersects the unit circle at point B in the first quadrant.

It forms an angle θ with the x-axis and a right-angled triangle with OB as its hypotenuse.

Using Pythagoras' theorem gives $(\sin\theta)^2 + (\cos\theta)^2 = 1^2$
or $\sin^2\theta + \cos^2\theta = 1$

If you find the gradient of the line using the points O(0, 0) and point B (cos θ, sin θ)

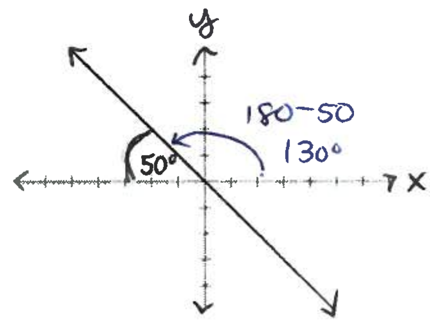
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sin\theta - 0}{\cos\theta - 0} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$



So these three properties are true for any angle θ

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
3. For any line $y = mx$ which forms an angle θ with the x-axis, the value of m (the gradient of the line) is $\tan\theta$.

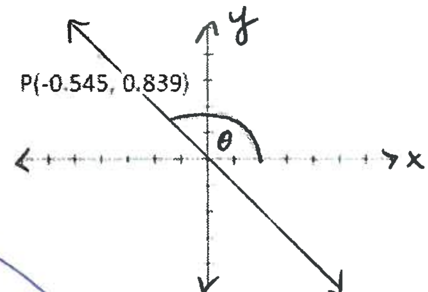
Example: Find the gradient of the line $y = mx$ in the diagram, giving your answer to three significant figures.
(Hint: θ must be an angle in standard position)



$$m = \tan\theta = \frac{\sin(130)}{\cos(130)}$$

$$\boxed{m = -1.19}$$

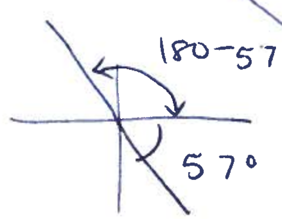
Example: Find the equation of the line passing through the origin and point P. Find the value of θ to the nearest degree.



$$m = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\cancel{\sin\theta}\ .839}{\cancel{\cos\theta}\ -.545} = -1.54$$

$$\tan^{-1}\left(\frac{.839}{-.545}\right) = \theta'$$

$$\theta' = -57°$$



$$\boxed{\theta = 123°}$$

$y = mx$

$$\boxed{y = -1.54x}$$

Exercise 11 F