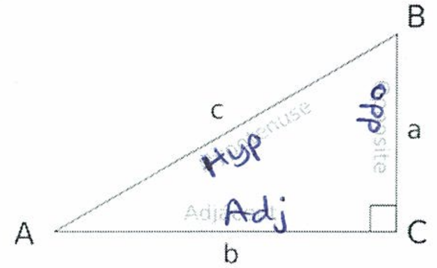
Section 11.1 Right-angled Triangle Trigonometry

For a given ACUTE angle in a RIGHT triangle the following ratios exist:

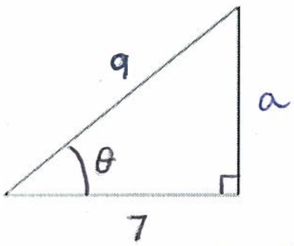


$$\sin \hat{A} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \hat{A} = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan \hat{A} = \frac{\text{opposite}}{\text{adjacent}}$$

Remember that in IB notation, $\angle A$ is the same as $\hat{A}$

Example: Write the 3 trig ratios that correspond to the triangle below

First find the hypotenuse using the Pythagorean Theorem $a^2 + b^2 = c^2$.
Remember the hypotenuse is ALWAYS c.



$a^2 + 7^2 = 9^2$

$a = \sqrt{32} \quad a = 4\sqrt{2}$

$\sin = \frac{4\sqrt{2}}{9}$

$\cos = \frac{7}{9}$

$\tan = \frac{4\sqrt{2}}{7}$

Reduce or rationalize if needed

Remember from Precalculus

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

(see proof pg 356)

If you know the ratio, you can find the corresponding angle by using the appropriate inverse trigonometric function of $\sin^{-1}\theta$, $\cos^{-1}\theta$, or $\tan^{-1}\theta$ (also called arc sine, arc cosine, or arc tangent)

Example: Use the calculator to find the following measures (in DEGREE mode).
Give answers correct to 3 significant figures where necessary

$\sin 27° \approx$ .454

$\tan 145° \approx$ −.700

$\cos\theta = \frac{3}{4}$ $\cos^{-1}(3/4)$ 41.4°

$\cos\theta = 2$ undefined!

Properties of 30-60-90 and 45-45-90 triangles can be used to find exact values of the trig ratios for 30°, 60°, and 45°. These angles are called special angles and their trig values should be memorized. When these angles are used, answers should always be given as exact unless otherwise stated (which means NO DECIMALS). Note: square roots of numbers that are not perfect squares are called surds.

How can we come up with those values without memorizing it totally…

Method 1: Make your own chart… to do this you must know that tan = sin/cos

	30	45	60
sin	$\sqrt{1} = 1$	$\sqrt{2}$	$\sqrt{3}$
cos	$\sqrt{3}$	$\sqrt{2}$	$\sqrt{1} = 1$
		Divide by 2	

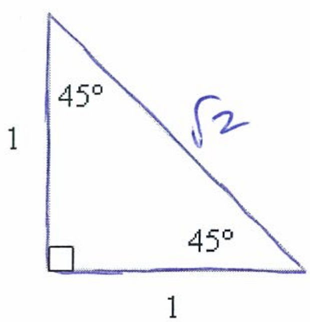
So $\sin 45° = \sqrt{2}$ divided by 2 or $\sqrt{2}/2$

That means $\tan 45° = \sin/\cos = \sqrt{2}/\sqrt{2} = 1$

Or you can use the hand trick (see https://www.youtube.com/watch?v=xXGfp9PKdXM)

Method 2: Special Right Triangles

45-45-90
(Isosceles Triangle)



$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

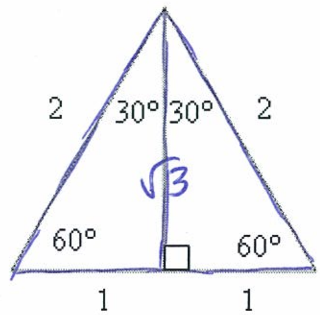
$\cos 45° = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{1}{1} = 1$

$1^2 + 1^2 = c^2$

$c = \sqrt{2}$

30-60-90
(Half an Equilateral Triangle)



$a^2 + 1^2 = 2^2$

$a = \sqrt{3}$

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

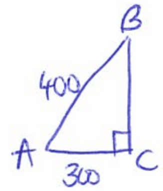
$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60° = \frac{\sqrt{3}}{2}$

etc…

Example: Find all the unknown angles and sides correct to three significant figures. Let $\hat{C}$ be the right angle. Use exact answers whenever possible.

1) $b = 300$, $c = 400$



$\sin B = \frac{300}{400}$

$\sin^{-1}(300/400)$

$\boxed{B = 48.6°}$

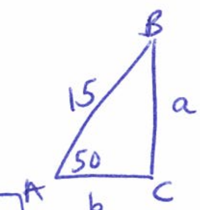
$\boxed{A = 41.4°}$

$a^2 + 300^2 = 400^2$

$a^2 = 70{,}000$

$\boxed{a = 100\sqrt{7} \text{ or } 265}$

2) $c = 15$, $\hat{A} = 50°$



$\sin 50 = \frac{a}{15}$

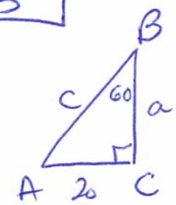
$\boxed{a = 11.5}$

$\cos 50 = \frac{b}{15}$

$\boxed{b = 9.64}$

$\boxed{B = 40°}$

3) $\hat{B} = 60°$, $b = 20$



$\boxed{A = 30°}$

$\sin 60 = \frac{20}{c}$

$\frac{\sqrt{3}}{2} = \frac{20}{c}$

$c = \frac{40}{\sqrt{3}} =$

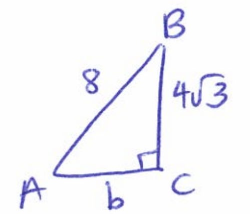
$\boxed{c = \frac{40\sqrt{3}}{3}}$

$\tan 30 = \frac{a}{20}$

$\frac{\sqrt{3}}{3} = \frac{a}{20}$

$\boxed{a = \frac{20\sqrt{3}}{3}}$

4) $a = 4\sqrt{3}$, $c = 8$



$\sin A = \frac{4\sqrt{3}}{8}$

$\sin A = \frac{\sqrt{3}}{2}$

$\boxed{A = 60°}$

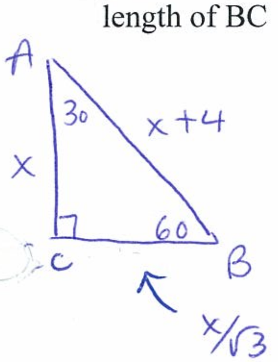
$\boxed{B = 30°}$

$\sin 30 = \frac{b}{8}$

$\frac{1}{2} = \frac{b}{8}$

$\boxed{b = 4}$

Example: Triangle ABC has $\hat{A} = 30°$, $\hat{C} = 90°$, AB $= x + 4$ and AC $= x$. Find the value of x and the length of BC



By special right triangles, $BC = x/\sqrt{3}$

$x^2 + (x/\sqrt{3})^2 = (x+4)^2$

$x^2 + \frac{x^2}{3} = x^2 + 8x + 16$

$3x^2 + x^2 = 3x^2 + 24x + 48$

$x^2 - 24x - 48 = 0$

$x = \frac{24 \pm \sqrt{768}}{2} = \frac{24 \pm 16\sqrt{3}}{2}$

can't be negative

$\boxed{x = 12 + 8\sqrt{3} \approx 25.9}$

$BC = 14.9$

or solve by graphing by finding x-intercepts

Exercise 11 A & B